Homework 4

Due 5/2/2019

1. Let $G = \{z \in \mathbb{C} : |z| < 2 \text{ and } \mathrm{Re}(z^2) \leq 1\}$.
   (a) Sketch the region G.
   Hint: use command **RegionPlot** of Mathematica.
   (b) Determine the interior points of G.
   (c) Determine the boundary points of G.
   (d) Determine whether G is open, or closed, or neither.

2. To each of the following functions, determine the region of continuity. That is, find the set of *all points* where the function is continuous. Make sure to justify your answers.
   (a) $f(z) = \bar{z}$
   (b) $f(z) = |z|$
   (c) $f(z) = \sinh z$
   (d) $f(z) = (z+1)^{1/2}$ (principal branch being used)
   (e) $f(z) = \mathrm{Log}(z-i) + \mathrm{Log}(z+i)$

3. Find a parametrization for each of the following curves:
   (a) the circle centered at $1+i$ with radius 3,
   (b) the line segment from $-1-i$ to $2i$,
   (c) the infinite line passing through $1-2i$ and $2+i$,
   (d) the upper half of the circle centered at $-1+i$ with radius 2 oriented clockwise.

4. Do Problem 2.18 (a), (b), (h) on page 32 of the textbook.

The purpose of the next problem is to use Mathematica to visualize the multivalued function $\log(z^3+1)$. *Make sure to write the Mathematica code you use, give explanation and some comments on the graph.* Similar treatment is done for function $\log(z^2+i)$ in the supplemental material "**Multivalued functions via Mathematica**" posted on the course website.

5. Consider the multivalued function $g(z) = \log(z^3+1)$.
   (a) By the definition of logarithm, what is the real part and imaginary part of $g(z)$?
   (b) Graph the real part of $g(z)$.
   (c) Call the imaginary part $f(z)$. Use the principal branch of the argument function (that is, Arg) to write the corresponding branch of $f(z)$. Call it $F(z)$.
   (d) Graph $F(z)$. Then compute $F(1+i)$ by hand.
   (e) Based on the graph of $F(z)$, describe geometrically the branch cut and branch point(s).
   (f) Compute these branch points (complex numbers written in standard or polar form).
   (g) Find the branch cut by solving for z's such that $z^3 + 1 \in \mathbb{R}_{\leq 0}$. Hint: locate z^3 on the complex plane, then use geometry to locate z.
   (h) Concatenate several branches of $f(z)$ to get a graph of $f(z)$. (For better visualization, use only 2 or 3 consecutive branches, for example $k = -1, 0, 1$.)