Homework 8
Due 6/7/2019

In Problem 3 and 4, you should use Mathematica to sketch the curves. You can use Mathematica to assist your arguments. *Make sure to write the Mathematica code you use, give explanation and some comments on the graph.*

1. Find the radius of convergence and the region of convergence of the following power series:

   (a) $$\sum_{n=1}^{\infty} \frac{2^n}{1+3^n} z^n$$

   (b) $$\sum_{n=1}^{\infty} \frac{z^n}{\left(\sqrt{3}+i\right)^n}$$

2. Find three different Laurent series representations (about 0) for the function

$$f(z) = \frac{3}{-z^2+z+2}$$

3. Let γ be a path with parametrization

$$\begin{cases} x(t) = 3\cos t \cos 3t, \\ y(t) = 3\sin t \cos 3t \end{cases} \qquad t \in [0, 2\pi]$$

   and function

$$f(z) = \frac{1}{(z^2+2z+2)(z-1)}$$

   (a) Sketch γ.

   (b) Compute the exact value of $\int_\gamma f(z)dz$. Write the result in complex standard form $a+ib$. *Hint:* split γ into three simple curves. Pay attention to the orientation of each curve. Use Cauchy's Integral formula for each curve.

In Problem 4, similar treatment is done in the supplemental material "**Evaluating complex integral by series**" posted on the course website.

4. Let γ be a curve with parametrization

$$\begin{cases} x(t) = \sin t + \sin 2t, \\ y(t) = \cos t + \cos 5t \end{cases} \qquad t \in [0, \pi]$$

   and function $f(z) = \frac{e^z}{z^3}$.

   (a) Sketch γ.

   (b) Find Laurent series representation of $f(z)$ about 0.

   (c) Use Part (b) to find an approximation for $\int_\gamma f(z)dz$. Roundup to four digits after decimal point.