Worksheet
5/3/2019

1. Where is the following function differentiable? Where is it holomorphic? Determine its derivative at points where it is differentiable.

$$f(z) = x^2 + y^2 + i2xy$$

2. Find all real constants a and b such that $f(z) = (2x - y) + i(ax + by)$ is an entire function.

3. Determine and sketch the region of continuity of the following complex functions.

(a) $\frac{z+1}{z^2+1}$

(b) $\sqrt{iz-1}$
Hint: write $z = x + iy$

(c) $\sqrt{z+1} + \sqrt{2z-i}$

4. Determine whether the following limits is a complex number, infinity or does not exist.

(a)

$$\lim_{z\to\infty} \frac{z+i}{iz+1}$$

Hint: Divide numerator and denominator by z.

(b)

$$\lim_{z\to\infty} \frac{1}{z-a}$$

where a is a given complex number.

(c)

$$\lim_{z\to 0} \frac{|z|^2}{z}$$

5. Let $f(z) = \frac{z^2}{|z|^2}$

(a) Find $\lim_{z\to 0} f(z)$ as $z \to 0$ along the line $y = x$.

(b) Find $\lim_{z\to 0} f(z)$ as $z \to 0$ along the line $y = 2x$.

(c) Does the limit $\lim_{z\to 0} f(z)$ exist?