Some review problems for Final Exam

1. Compute the following limits. Distinguish between the case the limit is equal to ∞ and the case the limit does not exist.

   (a)
   $$\lim_{z \to 0} \frac{\sin z - z}{z^3}$$

   (b)
   $$\lim_{z \to -1+i} \frac{z+1-i}{z(z^2+2z+2)}$$

   (c)
   $$\lim_{z \to i} \frac{e^{\pi z}+1}{z-i}$$

   (d)
   $$\lim_{z \to 0} e^{i\,\mathrm{Arg}(z^4)}$$

   (e)
   $$\lim_{z \to \infty} \frac{\mathrm{Log}\, z}{z}$$

2. To each of the following functions, determine the region where it it continuous/ differentiable/ holomorphic. Find the derivative at $z = 1+i$ if the function is differentiable at $i+1$.

   (a) $f(z) = \mathrm{Log}\left(\frac{z+1}{z-1}\right)$

   (b) $f(z) = \mathrm{Log}(iz^2)$

   (c) $f(z) = e^{ix}e^{iy}$

3. Evaluate the complex integrals $\int_\gamma f(z)$ where f and γ are given as follows. Clearly mention the method/theorem you use.

   (a) $f(z) = x^2 + iy$ and γ is the part of the curve $x = \sqrt{y}$ from $y = 4$ to $y = 1$.

   (b) $f(z) = \frac{1}{z^4+z^2+1}$ and γ is the rectangle with vertices at $(1,0)$, $(1,1)$, $(-1,1)$, $(-1,0)$ oriented in that order.

   (c) $f(z) = z\mathrm{Log}(z^2+1)$ and $\gamma(t) = \frac{1}{2}e^{-it}$ where $t \in [0,\pi]$.