## Homework 1

Due 4/8/2020

1. Write the following complex numbers in standard form $a+i b$.
(a) $(1+2 i)(2-i)+(1+i)^{2}$
(c) $\frac{2+4 i}{1+i}+(1+2 i)^{2} i^{5}$
(b) $(2-i)^{4}$
(d) $\frac{|-1-2 i|}{3+i \sqrt{2}}$
2. Write the following complex numbers in polar form $r e^{i \theta}$.
(a) $\frac{1}{1+i}+\frac{1}{-1+i}$
(c) Third roots of $1+\sqrt{3} i$
(b) $-2+i \sqrt{12}$
(d) Fourth roots of $\frac{1}{2}-\frac{\sqrt{3}}{2} i$
3. Find all $z \in \mathbb{C}$ that satisfy the following equation.
(a) $|z|-2 z=i$
(b) $z^{2} \bar{z}=z$
(c) $z^{2}+(1+2 i) z+i-7=0$
(d) $z^{6}+z^{3}+1=0$
(e) $z^{3}+(2-i) z^{2}+(2-2 i) z-2 i=0$

Hint: $z=i$ is one of the roots.
4. Sketch the following sets on the complex plane. You can either sketch by hand or use Mathematica to plot (see the instruction file posted on Canvas or course website). If you use Mathematica, make sure to write on your homework the command(s) you use to plot.
(a) $\operatorname{Re}\left(z^{2}\right)>0$
(b) $0<|z-1|<2$
(c) $|z| \leq|z-4|$
(d) $|z+1|>|z|$ and $1<|z|<2$
(e) $\operatorname{Re}(z)>\operatorname{Im}(z)$ and $|z-1|<1$

