Homework 5

Due 5/15/2020

Before starting this homework, please take a look at the supplemental material called "**Discontinuity and Angles**" posted on Canvas and course website. *Make sure to include the Mathematica codes and figures you use and some brief comments.*

1. Let
$$f(z) = \left(\frac{z+1}{z-1}\right)^i$$
   (a) Determine the region of continuity of f. That is, find *all* $z \in \mathbb{C}$ where f is continuous.
   (b) Pick a point where f is discontinuous. Call it z_0. Use Mathematica to describe how f jumps at z_0.
   Hint: draw a curve that passes through z_0. Draw the image of this curve under f.

2. Determine the region where each of the following function is differentiable. Find the derivative of the function. Determine the region where the function is holomorphic.
   (a) $f(z) = \bar{z}^2 z$
   (b) $f(z) = x^2y + x + i(xy^2 - x + y)$
   where $z = x + iy$.
   (c) $f(z) = i^z$ (principal logarithm is used)

3. Determine the region where the function $f(z) = \tan z$ is differentiable. Then show that $f'(z) = 1 + \tan^2 z$ in this region.

4. Let $f(z) = \frac{z^2}{z-3i}$ and $z_0 = 0$. Use Mathematica to show that f is not angle-preserving at z_0. What does f do to the angles at z_0 instead?

5. Consider the function $f(z) = z^3$.
   (a) Sketch the image of the vertical line (ℓ): $x = 1$ under f. Note that the image curve intersects itself.
   (b) Find two distinct points z_1 and z_2 on (ℓ) such that $f(z_1) = f(z_2)$.
   Hint: $(\frac{z_1}{z_2})^3 = 1$. Find $\operatorname{Arg} z_1$ and $\operatorname{Arg} z_2$. Then use geometry to determine z_1 and z_2.
   (c) Find $f'(z_1)$ and $f'(z_2)$.
   (d) Find the angle at which the image curve intersects itself. Round to 4 digits after the decimal point.