73 points total

# Group work 4

Due 2/8/2019

**Name:** ______________________________

**Instructions:** Show your work. In problems that require Mathematica, write one or two commands that best illustrate how you get the answer. You are not required to write all the codes. Circle your final answers. The assignment has 6 pages.

1. Use substitute (the change of variables) to find the following indefinite integrals. Your answer should contain an undetermined constant.

   (i) $\int 2x(x^2+1)^4 dx$

   $f(x) =$ $2x(x^2+1)^4$

   $u =$ $x^2+1$

   4

   $du =$ $2x\,dx$

   $f(x)dx =$ $u^4\,du$

   $\int f(x)dx =$ $\int u^4 du = \frac{u^5}{5} + C = \frac{(x^2+1)^5}{5} + C$

   (ii) $\int \frac{8x+6}{2x^2+3x} dx$

   $f(x) =$

   $u =$ $2x^2+3x$

   4

   $du =$ $(4x+3)dx$

   $f(x)dx =$ $\frac{1}{2u}\,du$

   $\int f(x)dx =$ $\int \frac{1}{2u} du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|2x^2+3x| + C$

(iii) $\int 8x\cos(4x^2+3)dx$

$f(x) =$

$u =$ $4x^2+3$ 4

$du =$ $8x\,dx$

$f(x)dx =$ $\cos(u)\,du$

$\int f(x)dx =$ $\int \cos u\,du = \sin u + C = \sin(4x^2+3)+C$

(iv) $\int \frac{\sin x}{\cos^3 x}dx$

$f(x) =$

$u =$ $\cos x$

$du =$ $-\sin x\,dx$ 4

$f(x)dx =$ $-\frac{du}{u^3}$

$\int f(x)dx =$ $\int -\frac{1}{u^3}du = \frac{1}{2}u^{-2}+C = \frac{1}{2\cos^2 x}+C$

(v) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$f(x) =$

$u =$ $e^x + e^{-x}$

4

$du =$ $(e^x - e^{-x})dx$

$f(x)dx =$ $\frac{du}{u}$

$\int f(x)dx =$ $\int \frac{1}{u} du = \ln|u| + C = \ln(e^x + e^{-x}) + C$

2. This exercise aims to find the length of the following curve:

$$y = \frac{(x^2+2)^{3/2}}{3}, \quad x \in [0,1]$$

(i) Review: what is the definition for the length of the curve $y = f(x)$ on $x \in [a,b]$?

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

3

(ii) In this problem, what is $f(x)$ and what is the interval $[a,b]$?

$$f(x) = \frac{(x^2+2)^{3/2}}{3}, \quad [a,b] = [0,1]$$

2 2

(iii) Find $f'(x)$.

$$f'(x) = \frac{3}{2}\, 2x\, \frac{1}{3} (x^2+2)^{1/2} = x\,(x^2+2)^{1/2}$$

3

(iv) Find $1 + f'(x)^2$.

$1 + x^2(x^2+2)$

3

(v) Simplify $\sqrt{1 + f'(x)^2}$.

$\sqrt{1 + x^4 + 2x^2} = \sqrt{(1+x^2)^2} = 1+x^2$

3

(vi) Evaluate $L$.

$L = \int_0^1 (1+x^2)\,dx = \left(x + \frac{x^3}{3}\right)\Big|_0^1 = 1 + \frac{1^3}{3} = \frac{4}{3}$

3

3. A cyclist is riding on a road with a slight slope uphill. Suppose the initial velocity (at time $t = 0$) is 400 m/min. He decelerates at 20 m/min$^2$.

(i) What is the acceleration function $a(t)$?

$a(t) = -20$ $(m/min^2)$

3

(ii) What is the velocity function $v(t)$?

$v(t)$ is an antiderivative of $a(t)$.

3

$v(t) = -20t + C$

Because $v(0) = 400$, $C = 400$.

$\boxed{v(t) = -20t + 400}$

(iii) Use the initial position at reference (that is, assume $s(0) = 0$). What is the position function $s(t)$?

$s(t)$ is an antiderivative of $v(t)$:

$$s(t) = -10t^2 + 400t + C$$

3

Because $s(0) = 0$, $C = 0$.

$$\boxed{s(t) = -10t^2 + 400t}$$

(iv) Find the average speed of the cyclist in the first 10 min.

$$\frac{1}{10}\int_0^{10} |v(t)|\,dt = \frac{1}{10}\int_0^{10} |\underbrace{400 - 20t}_{\geq 0}|\,dt = \frac{1}{10}\int_0^{10} (400 - 20t)\,dt$$

3

$$= \frac{1}{10}\left(400t - 10t^2\right)\Big|_0^{10} = \frac{1}{10}(4000 - 1000) = \boxed{300} \ (m/min)$$

(v) Find the position of the cyclist at $t = 25$ min.

$$s(25) = -10(25)^2 + 400(25) = 3750 \ (m)$$

3

(vi) What is the distance traveled in the first 30 min.

$$\int_0^{30} |v(t)|\,dt = \int_0^{30} |400 - 20t|\,dt = \int_0^{20} (400 - 20t)\,dt + \int_{20}^{30} (20t - 400)\,dt$$

$$= \left(400t - 10t^2\right)\Big|_0^{20} + \left(10t^2 - 400t\right)\Big|_{20}^{30}$$

4

$$= (8000 - 4000) + (-3000 + 4000)$$

$$= 5000 \ (m)$$

4. For most functions $f(x)$, it is hard to find an antiderivative, for example $f(x) = \cos(x^2)$, $e^{x^2}$, $\sqrt{1 + x^3}$,... In such a case, in order to compute definite integral of $f(x)$, one needs to use a numerical method. We already practiced left, right, mid-point Riemann sums. This time we will use Trapezoid sum, which is equivalent to a general Riemann sum, to approximate the

following definite integral:

$$\int_{-1}^{2} \cos(x^2)dx$$

(i) The interval $[-1, 2]$ is partitioned into $n = 300$ equal subintervals $[x_0, x_1]$, $[x_1, x_2]$,..., $[x_{n-1}, x_n]$. What is the width of each subinterval?

$$\frac{3}{300} = \frac{1}{100}$$

3

(ii) Express $x_k$ in terms of $k$.

$x_0 = -1$

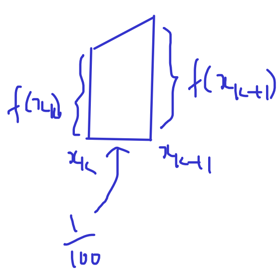
$x_1 = -1 + \frac{1}{100}$

$x_2 = -1 + \frac{2}{100}$

$x_k = -1 + \frac{k}{100}$

3

(iii) Express the area of the trapezoid on the subinterval $[x_k, x_{k+1}]$ in terms of $k$.




$$\frac{1}{2}\left(f(x_k) + f(x_{k+1})\right)\frac{1}{100}$$

3

$$= \frac{1}{2}\left[\cos\left(\left(-1+\frac{k}{100}\right)^2\right) + \cos\left(\left(-1+\frac{k+1}{100}\right)^2\right)\right]\frac{1}{100}$$

(iv) Write the trapezoid sum (using the sigma summation notation) that approximates the given definite integral.

$$\sum_{k=0}^{299} \frac{1}{2}\left[\cos\left(\left(-1+\frac{k}{100}\right)^2\right) + \cos\left(\left(-1+\frac{k+1}{100}\right)^2\right)\right]\frac{1}{100}$$

3

(v) Use Mathematica to evaluate this sum.

Sum[ $\frac{1}{2}$ * ( Cos[ (-1 + $\frac{k}{100}$)^2 ] + Cos[ (-1 + $\frac{k+1}{100}$)^2 ] ) * $\frac{1}{100}$ , {k, 0, 299} ]

$\boxed{1.366}$

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