Worksheet
3/15/2019

1. Evaluate the area of the region bounded by the curves $y = x$, $y = x^2 - 2$, $y = 0$ in the third quadrant.

2. Evaluate the length of the arc

$$y = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}, \qquad x \in [0, 9]$$

3. Check if the following integral converges or diverges. If it converges, evaluate it.

$$\int_1^\infty \frac{dx}{x^2(x+1)}$$

4. Solve the following initial value problem:

$$\frac{dy}{dx} = 2y(y-3), \qquad y(0) = 1$$

5. An initial deposit of \$1500 is put in a saving account. Suppose the interest is compounded continually at the rate of 0.1% per year.

   (a) Find the balance in the account as a function of time.

   (b) How long does it take for the balance to increase in value to \$2500?

6. A cup of hot water at 140°F is left in a room of temperature 72°F. The heat constant is $k = 0.1$ (min^{-1}).

   (a) Use Newton's law of cooling to find the temperature of the water in the cup as a function of time (minutes).

   (b) How long does it take for the temperature to be 90°F?

7. A tank contains 1000 gallons of mixture of salt and water. The initial amount of salt is 10 lbs. People purify the mixture by pumping in fresh water at a rate of 5 gal/min, while at the same time pumping the mixture out at a rate of 4 gal/min. Find the amount of salt (lbs) in the tank as a function of time.

8. Use an appropriate method to evaluate the following integrals.

   (a) $\int \frac{3x}{\sqrt{x+4}} dx$ (Hint: use substitution.)

   (b) $\int \sin^3 x \cos^3 x dx$ (Hint: use substitution.)

(c) $\int_{-2}^{1} \frac{3}{x^2+4x+13} dx$ (Hint: complete the square in the denominator, then use substitution.)

(d) $\int_0^1 \frac{t}{e^{2t}} dt$ (Hint: use integration by part.)