Homework 2

1. Check if each following map is a linear map. If it is, explain why (by verifying the 2 criteria). If it is not, show how one of these criteria is violated.

   (a) $f : \mathbb{R}^2 \to \mathbb{R}^2$, $f(x_1, x_2) = (x_1 + x_2,\, x_1 - x_2)$.

   (b) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x_1, x_2) = x_1^2 + x_2^2$.

   (c) $f : \mathbb{R}^3 \to \mathbb{R}$, $f(x_1, x_2, x_3) = x_1 x_2 x_3$.

2. Let

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & 3 \\ -3 & 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \\ -1 & 4 & -3 \end{bmatrix}$$

   Compute $(2A - B)^2$.

3. Recall that *zero matrix* is a matrix whose every entry is equal to 0. For convenience, an $m \times n$ zero matrix is often denoted as 0 (as if it were the number zero). The size of the matrix is usually understood in the context.

   Give an example of a 2-by-2 nonzero matrix A such that $A^2 = 0$.

4. Determine (i.e. write the formula of) a linear map $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(1,\, 2) = 1$ and $f(2,\, 5) = 4$.

5. Let

$$A = \begin{bmatrix} 2 & 0 & 1 & 3 \\ 1 & -3 & 4 & 0 \\ -1 & -4 & 3 & -2 \end{bmatrix}$$

   Find the linear map associated with A. (This includes finding the domain, the target set, and an explicit formula of f).

6. Find the matrix associated with the following linear map:

   (a) $f : \mathbb{R}^2 \to \mathbb{R}^2$, $f(x_1, x_2) = (2x_1 - x_2,\, 0)$.

   (b) $f : \mathbb{R}^3 \to \mathbb{R}^2$, $f(x_1, x_2, x_3) = (x_1 - x_2 - x_3,\, x_2)$.

   (c) $f : \mathbb{R} \to \mathbb{R}^2$, $f(x) = (2x,\, -x)$.

7. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}^3$ be linear maps given by

$$\begin{aligned} f &: \mathbb{R}^2 \to \mathbb{R}^2, & f(x_1, x_2) &= (2x_2,\, x_1), \\ g &: \mathbb{R}^2 \to \mathbb{R}^3, & g(x_1, x_2) &= (x_2,\, x_1 - x_2,\, x_1). \\ h &: \mathbb{R}^2 \to \mathbb{R}^2, & h(x_1, x_2) &= (0,\, 3x_2 - 2x_1). \end{aligned}$$

   What are the matrices associated with f, g and h? To each of the following maps, first write an *explicit formula*, then find the *associated matrix*:

   (i) $f + h$

   (ii) $f - 2h$

   (iii) $g \circ f$

8. Do Problems 1, 5, 7 of Section 3.8 (page 53) of the textbook by using Gauss elimination method.