Homework 3

1. Solve the following systems of linear equations using Gauss elimination method. If a system has infinitely many solutions, write them in parametric vector form.

(a)

$$\begin{cases} 6x - 2z &= 8 \\ x + 2y &= 5 \\ -y + 3z &= -5 \end{cases}$$

(b)

$$\begin{cases} 2x - y + z &= 0 \\ x + 2y - 2z &= 0 \\ 3x + y - z &= 1 \end{cases}$$

(c)

$$\begin{cases} x + 2y + 3z &= 14 \\ 3x + 2y + z &= 10 \\ 3x + y + 2z &= 11 \end{cases}$$

(d)

$$\begin{cases} x + y + 2z &= 4 \\ x - 2y + z &= 0 \\ x - 5y &= -4 \end{cases}$$

(e)

$$\begin{cases} x_1 - 8x_3 + 7x_4 &= 0 \\ x_1 + x_2 - 2x_3 + 2x_4 &= 0 \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 &= 0 \end{cases}$$

2. (Problem 8, page 92 textbook) Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear map associated with matrix

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 2 & -8 & 8 \end{bmatrix}$$

Solve the equation $f(x) = (-2,\, 12)$. (That is, find all vectors $x = (x_1,\, x_2,\, x_3) \in \mathbb{R}^3$ such that $f(x_1, x_2, x_3) = (-2,\, 12)$.)

3. (Problem 9, page 92 textbook) Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear map associated with matrix

$$A = \begin{bmatrix} 1 & 2 \\ -3 & -8 \\ 1 & 8 \end{bmatrix}$$

Solve the equation $f(x) = (-2,\, 12,\, -20)$.

4. Find the reduced row echelon form (RREF) of the following matrices. Make sure to show each row operation step.

(a)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & -1 \\ -2 & -1 & -2 \end{bmatrix}$$

5. Check if each following matrix is invertible. If so, find the inverse matrix.

(a)

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & -1 \\ -2 & -1 & -2 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 2 & 2 & 3 \\ -1 & 0 & 1 \\ 0 & 2 & 5 \end{bmatrix}$$

6. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map given as follows:

$$f(x, y, z) = (x + y + z,\ 6x + 5y + 4z,\ 13x + 10y + 8z).$$

(a) Find the matrix A associated with f.

(b) Find A^{-1}.

(c) Find an explicit expression for f^{-1} (the inverse map of f).