

Solution Problem 1f:

$$f(x) = \sqrt[3]{1-x^2} = (1-x^2)^{\frac{1}{3}} \quad \dots\dots\dots f(3) = \sqrt[3]{-8} = -2$$
$$f'(x) = \frac{1}{3}(-2x)(1-x^2)^{-\frac{2}{3}} \quad \dots\dots\dots f'(3) = -2(-8)^{-\frac{2}{3}} = (-2)(-2)^{-2} = -\frac{1}{2}$$

$$f''(x) = -\frac{2}{3}(1-x^2)^{-\frac{2}{3}} + \frac{1}{3}(-2x)\left(-\frac{2}{3}\right)(-2x)(1-x^2)^{-\frac{5}{3}}$$

$$f''(3) = -\frac{2}{3}(-8)^{-\frac{2}{3}} - 8(-8)^{-\frac{5}{3}}$$
$$= -\frac{2}{3} \frac{1}{4} - 8\left(-\frac{1}{32}\right)$$
$$= \frac{1}{12}$$

Rewrite  $f''(x) = -\frac{2}{3}(1-x^2)^{-\frac{2}{3}} - \frac{8}{9}x^2(1-x^2)^{-\frac{5}{3}}$

$$f'''(x) = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)(1-x^2)^{-\frac{5}{3}} - \frac{8}{9}2x(1-x^2)^{-\frac{5}{3}} - \frac{8}{9}x^2\left(-\frac{5}{3}\right)(-2x)(1-x^2)^{-\frac{8}{3}}$$
$$= \frac{4}{9}(1-x^2)^{-\frac{5}{3}} - \frac{16}{9}x(1-x^2)^{-\frac{5}{3}} - \frac{80}{27}x^3(1-x^2)^{-\frac{8}{3}}$$

$$f'''(3) = \frac{4}{9}(-8)^{-\frac{5}{3}} - \frac{16}{3}(-8)^{-\frac{5}{3}} - 80(-8)^{-\frac{8}{3}}$$
$$= \frac{4}{9}\left(-\frac{1}{32}\right) - \frac{16}{3}\left(-\frac{1}{32}\right) - 80\left(\frac{1}{256}\right)$$
$$= -\frac{23}{144}$$

Therefore,  $T_3(x) = f(3) + \frac{f'(3)}{1}(x-3) + \frac{f''(3)}{2}(x-3)^2 + \frac{f'''(3)}{6}(x-3)^3$

$$= -2 - \frac{1}{2}(x-3) + \frac{1}{24}(x-3)^2 - \frac{23}{864}(x-3)^3$$