Solution to some prob. of HW7

1.b) $\sum \underbrace{\frac{3+\cos n}{n^2}}_{a_n}$

Notice: $a_n \geq 0$.

$$a_n \leq \frac{3+1}{n^2} = \frac{4}{n^2} = b_n$$

the series $\sum \frac{4}{n^2} = 4\sum \frac{1}{n^2}$ converges (p-series with $p=2$)

Therefore $\sum a_n$ converges by Comparison test.

1.c) $\sum \underbrace{\sin\left(\frac{n\pi}{4}\right)}_{a_n}$

a_n : $\underbrace{\frac{\sqrt{2}}{2}}_{a_1}, \underbrace{1}_{a_2}, \underbrace{-\frac{\sqrt{2}}{2}}_{a_3}, \underbrace{0}_{a_4}, \underbrace{-\frac{\sqrt{2}}{2}}_{a_5}, \underbrace{-1}_{a_6}, \underbrace{\frac{\sqrt{2}}{2}}_{a_7}, \underbrace{0}_{a_8}, \dots$

This sequence doesn't converge to 0 because

$$a_{4k+2} = \sin\left(\frac{(8k+2)\pi}{4}\right) = 1 \quad \text{for all } k \geq 1$$

Therefore, the series diverges.

1.e) $\sum \underbrace{\frac{(-1)^n}{n^2-n}}_{a_n}$

$|a_n| = \frac{1}{n^2-n}$ Put $b_n = \frac{2}{n^2}$

we want to show that $|a_n| \leq b_n$ for n large.

The inequality $|a_n| \leq b_n$ is equivalent to

$$\frac{1}{n^2-n} \leq \frac{2}{n^2}$$

which is equiv. to $n^2 \leq 2(n^2-n)$,

" " $n^2 \leq 2n^2-2n$,

" " $2n \leq n^2$,

" " $2 \leq n$,

which is true if $n \geqslant 2$.

Therefore, $\sum a_n$ converges by Comparison test.