

Solution to some prob. in HW9

2 c) Method 1:

$$\begin{aligned}\frac{1+x^2}{4-x^2} &= (1+x^2) \frac{1}{4-x^2} = (1+x^2) \frac{\frac{1}{4}}{1-\frac{x^2}{4}} = \frac{1}{4} (1+x^2) \frac{1}{1-\frac{x^2}{4}} \\ &= \frac{1}{4} (1+x^2) \left( 1 + \frac{x^2}{4} + \left(\frac{x^2}{4}\right)^2 + \left(\frac{x^2}{4}\right)^3 + \dots \right) \\ &= \frac{1}{4} (1+x^2) \left( 1 + \frac{x^2}{4} + \frac{x^4}{16} + \frac{x^6}{64} + \dots \right) \\ &= \frac{1}{4} \left[ 1 + x^2 \left(1 + \frac{1}{4}\right) + x^4 \left(\frac{1}{16} + \frac{1}{4}\right) + x^6 \left(\frac{1}{64} + \frac{1}{16}\right) + \dots \right] \\ &= \frac{1}{4} + \frac{5}{16} x^2 + \frac{5}{64} x^4 + \frac{5}{256} x^6 + \dots\end{aligned}$$

Method 2:

$$\begin{aligned}\frac{1+x^2}{4-x^2} &= \frac{5-(4-x^2)}{4-x^2} = \frac{5}{4-x^2} - 1 = \frac{5}{4} \frac{1}{1-\frac{x^2}{4}} - 1 \\ &= \frac{5}{4} \left( 1 + \frac{x^2}{4} + \left(\frac{x^2}{4}\right)^2 + \left(\frac{x^2}{4}\right)^3 + \dots \right) - 1 \\ &= \frac{1}{4} + \frac{5}{16} x^2 + \frac{5}{64} x^4 + \frac{5}{256} x^6 + \dots\end{aligned}$$

2. h) Method 1:

$$\begin{aligned}\cos^2 x &= \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots \right)^2 \\ &= \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots \right) \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots \right)\end{aligned}$$

$$\begin{aligned}
&= 1 + x^2 \left( -\frac{1}{2} - \frac{1}{2} \right) + x^4 \left( \frac{1}{24} + \left( -\frac{1}{2} \right) \left( -\frac{1}{2} \right) + \frac{1}{24} \right) \\
&\quad + x^6 \left( -\frac{1}{720} - \frac{1}{2} \frac{1}{24} + \frac{1}{24} \left( -\frac{1}{2} \right) - \frac{1}{720} \right) + \dots
\end{aligned}$$

Method 2:

$$\begin{aligned}
\cos^2 x &= \frac{1 + \cos 2x}{2} = \frac{1}{2} + \frac{1}{2} \cos 2x \\
&= \frac{1}{2} + \frac{1}{2} \left( 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{24} - \frac{(2x)^6}{720} + \dots \right) \\
&= 1 - x^2 + \frac{1}{2} \frac{16}{24} x^4 - \frac{1}{2} \frac{64}{720} x^6 + \dots
\end{aligned}$$