

Homework 9

1. Do the problems 4 through 11 on page 198 (Section 16.6) of the textbook.
2. Use the following identities

$$\begin{aligned}\frac{1}{1-t} &= 1 + t + t^2 + t^3 + t^4 + \dots \quad (-1 < t < 1) \\ e^t &= 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \dots \\ \sin t &= t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \dots \\ \cos t &= 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \dots \\ \ln(1+t) &= t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \dots \quad (-1 < t < 1)\end{aligned}$$

to write the first 4 nonzero terms of the Taylor series (about 0) of the following functions.

(a) $\frac{1}{1-x^3}$

(b) $\frac{x^2+x}{1-x}$

(c) $\frac{1+x^2}{4-x^2}$

(d) $\cosh x = \frac{e^x + e^{-x}}{2}$

(e) $\ln\left(\frac{1+x}{1-x}\right)$

Hint: use the property $\ln(a/b) = \ln a - \ln b$.

(f) $x \sin\left(\frac{x}{2}\right)$

(g) $e^x \sin x$

(h) $\cos^2 x$

Hint: you can either square the Taylor series of cosine or use the trigonometric identity $\cos(2x) = 2\cos^2 x - 1$.