Some review problems for Final

1. Review Homework 5, 6, 7, 8.
2. Review all lecture worksheets starting from Worksheet 7.
3. Review recitation worksheets 6, 7, 8, 9, 10.
4. Let $G : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ be a linear map given by $G(u)(x) = (x+1)u'(x) - u(x)$. Is G diagonalizable? If it is, find a basis of $P_2(\mathbb{R})$ in which G is represented by a diagonal matrix.
5. Let $V = P_2(\mathbb{C})$. Show that the operator $(\cdot, \cdot)$ given by

   $$(u, v) = u(0)\,\overline{v(0)} + u(1)\,\overline{v(1)} + u(2)\,\overline{v(2)} \qquad \forall\, u, v \in V$$

   is an inner product on V.
6. On $M_{2\times 2}(\mathbb{R})$, consider an operator $\|\cdot\|$ given by $\|A\| = |a| + |b|$ for all $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Is $\|\cdot\|$ a norm on $M_{2\times 2}(\mathbb{R})$?
7. Let $V = M_{2\times 2}(\mathbb{C})$. The inner product on V is given by

   $$(A, B) = a_{11}\overline{b_{11}} + a_{12}\overline{b_{12}} + a_{21}\overline{b_{21}} + a_{22}\overline{b_{22}} = \operatorname{trace}(B^*A).$$

   Consider a linear map $f : V \to V$ given by $f(A) = AC - CA$ where

   $$C = \begin{bmatrix} 0 & i \\ 1 & 0 \end{bmatrix}.$$

   Find f^*. Is f an unitary map?