Homework 8

Due 03/13/2020

In the following problems, make sure to write your arguments coherently in full sentences. Start a sentence with words rather than a formula. Use words to transition your ideas, for example "This leads to", "Therefore", "We want to show", etc.

1. Find a least squares solution to the following inconsistent system $AX = b$.

   (a) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}$.

   (b) $A = \begin{bmatrix} i & 1 \\ 2 & -2i \end{bmatrix}$ and $b = \begin{bmatrix} i+1 \\ 1 \end{bmatrix}$.

2. Find a plane $z = ax + by + c$ that best fits five points $(1,1,1)$, $(1,2,4)$, $(2,0,1)$, $(-1,2,7)$, $(5,4,1)$. Is the answer unique?

3. Let $A, B \in M_{n\times n}(\mathbb{C})$. Recall that A is *unitary* if $A^{-1} = A^*$. Show that if A and B are unitary then so is AB.

4. Let $A \in M_{m\times n}(\mathbb{C})$. Show that

   (a) The eigenvalues of A^*A are real and non-negative.

   (b) $\text{null}(A^*A + I_n) = \{0\}$.

   (c) The matrix $A^*A + I_n$ is invertible.

5. Consider the real inner product space $V = P_2(\mathbb{R})$ with inner product given by

$$(u, v) = \int_{-1}^{1} u(x)v(x)dx.$$

   (a) Find an orthonormal basis of V.

   (b) Consider the derivative operator $D : V \to V$ given by $D(u) = u'$. Determine the adjoint operator D^*.

Do the following problem for 6 bonus points.

6. Find a singular value decomposition of the following matrices.

   (a) $A = \begin{bmatrix} 3 & -2 \\ -6 & -1 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

   (c) $C = \begin{bmatrix} 2 & 1-i \\ 1+i & 1 \end{bmatrix}$.