Some review problems for Midterm

1. Review Homework 1, 2, 3.
2. Review Worksheets 1 through 6.
3. Review Recitation worksheets.
4. Do Problem 1, Part (a) of Homework 4.
5. Consider the linear map $G : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ given by $G(u) = xu' - u$. Is G a monomorphism, epimorphism, isomorphism or none of them? Explain your answer.
6. Consider a linear map $f : M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ given by $f(A) = A^T + A$.
   (a) Find a matrix representation of f.
   (b) Consider
   $$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2\times 2}(\mathbb{R}) : \quad a + 2b = 0 \right\}.$$
   Is V invariant under f? Explain your answer.
7. Let
   $$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -2 \\ 1 & 0 & 3 \end{bmatrix}.$$
   Put
   $$\begin{aligned} V_1 &= \{v \in \mathbb{R}^3 : \ Av = v\}, \\ V_2 &= \{v \in \mathbb{R}^3 : \ Av = 2v\}, \\ V_3 &= \{v \in \mathbb{R}^3 : \ Av = 3v\}. \end{aligned}$$
   Show that $V_1 \oplus V_2 \oplus V_3 = \mathbb{R}^3$.