Worksheet
2/3/2020

Name: ____________________________________

1. Consider a linear map $G : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ given by $G(u) = u + (x+1)u'$. Which of the following spaces is invariant under G?

   (a) $W_1 = \{u \in P_2(\mathbb{R}) : \ u(-1) = 0\}$,

   (b) $W_2 = \{u \in P_2(\mathbb{R}) : \ u(0) = 0\}$.

(a) Let $u \in W_1$. We will check if $G(u)$ has to be in W_1

Let us write $v = G(u)$. To check if $v \in W_1$, we need to check if $v(-1) = 0$. We have

$$v = G(u) = u + (x+1)u'.$$

For $x = -1$,

$$v(-1) = u(-1) + \underbrace{(-1+1)}_{=0} u'(-1) = u(-1).$$

Because $u \in W_1$, we have $u(-1) = 0$. Thus, $v(-1) = 0$.

This means $v \in W_1$. We conclude that W_1 is invariant under f.

(b) Let $u \in W_2$. We will check if $G(u)$ has to be in W_2

Let us write $v = G(u)$. To check if $v \in W_2$, we need to check if $v(0) = 0$. We have

$$v = G(u) = u + (x+1)u'.$$

For $x = 0$,

$$v(0) = u(0) + (0+1)u'(0) = u(0) + u'(0).$$

Because $u \in W_2$, we have $u(0) = 0$. Thus, $v(0) = u'(0)$.

All we know about u is that it is a polynomial such that $u(0) = 0$.

This doesn't guarantee that $u'(0)=0$. In fact, we can give an explicit example: $u(x)=x$.

This function belongs to W_2. However, $G(u)$ doesn't belong to W_2 because

$$v(0) = G(u)(0) = u'(0) = 1 \neq 0.$$

We conclude that W_2 is <u>not</u> invariant under W_2.

2. Let V be a vector space over F, and $f : V \to V$ be a linear map. Let $\lambda \in F$. Show that the set $E = \{v \in V : \; f(v) = \lambda v\}$ is a subspace of V and that it is invariant under f.

Proof given in Lecture 13