Some review problems for Final

1. Review homework sets 5, 6, 7, 8. You can exclude all problems that involve coding.
2. Review all the worksheets after the midterm exam.
3. Consider the function $f(x) = \frac{5}{8(1+x^2)}$.
   (a) Solve for all fixed points of f.
   (b) Write an iteration formula for the fixed point method.
   (c) With $x_0 = 1$, what is the limit of (x_n)? Find the order of convergence. If the order of convergence is 1, find the linear rate of convergence.
   (d) Sketch a cobweb diagram that illustrates the fixed point method with $x_0 = 1$.
4. Find the interpolation polynomial of the following points using Lagrange and Newton formula:
   (a) $(-1, 1)$, $(0, -1)$, $(1, 1)$, $(2, 0)$.
   (b) $(-1, 0)$, $(0, -1)$, $(1, 0)$, $(0, 1)$.
5. Let f be a function such that $f(1) = 0$, $f(2) = 1$, $f(3) = -1$, $f(4) = 2$. Find the divided difference $f[1, 2, 3, 4]$.
6. Let $f(x) = \frac{1}{x^2-1}$. For evenly spaced sample points $3 = x_1 < x_2 < \ldots < x_n = 5$, let P_n be the corresponding interpolation polynomial. Find n such that
$$|f(x) - P_n(x)| \leq 10^{-4} \quad \forall\, x \in [3, 5].$$
7. Find a quadratic spline that fits three points $(-1, 1)$, $(0, 2)$, $(2, 0)$. Sketch this spline.
8. We want to find an approximate value of the integral $I = \int_1^2 \frac{1}{(x^2+1)^2} dx$. Let n be the number of equal subintervals of the interval $[1, 2]$.
   (a) For $n = 4$, use right-point rule to approximate I.
   (b) For $n = 4$, use midpoint rule to approximate I.
   (c) For $n = 4$, use trapezoid rule to approximate I.
   (d) How big should n be such that the midpoint rule gives an approximate value of I with error less than 10^{-4} ?