

Homework 5

Due 02/21/2020

1. Consider the function $f(x) = \frac{1}{4}(5 - x^2)$.
 - (a) Solve for all fixed points of f .
 - (b) Write an iteration formula for the fixed point method.
 - (c) With $x_0 = 0.8$, use your pocket calculator to guess the limit of the sequence (x_n) . Then use the iteration formula to verify that this number is truly a limit of (x_n) .
 - (d) Sketch a cobweb diagram that illustrates the fixed point method with $x_0 = 0.8$. The applet found on this website <https://www.geogebra.org/m/QJ79IWCL> can be helpful.
 - (e) Find the order of convergence. If the order of convergence is 1, find the linear rate of convergence.
2. In this problem, we will approximate the *largest* root of the function $f(x) = xe^x + \cos x$.
 - (a) Use Matlab to sketch the graph of function f .
 - (b) Use Newton's method to approximate the largest root of f . You will need to answer questions such as: what should x_0 be? What is the iteration formula? You can use an artificial criterion to stop the iteration such as: *the iteration will stop at step n if $|x_n - x_{n-1}| < 10^{-4}$.*
 - (c) Use fixed point method to approximate the largest root of f . You will need to answer questions such as: what is a function g whose fixed points are roots of f ? What should x_0 be? What is the iteration formula? You can use the same stopping criterion as in Part (b).

Hint: the choice of g is important. Some choices may not work. You can use the applet mentioned in Problem 1 to test your guesses.
3. Recall a general result: *Let α be a root of a function f . Suppose $f'(\alpha) \neq 0$. Then the order of the convergence of the Newton's method is 2.* In this problem, we will see that if $f'(\alpha) = 0$ then the order of convergence may drop. Consider the function $f(x) = x^2$.
 - (a) Sketch a diagram that illustrates the Newton's method with $x_0 = 1$.
 - (b) Write the iteration formula of the Newton's method.
 - (c) What is the limit of the sequence (x_n) ? What is the order of convergence?
4. In this problem, we want to find a polynomial curve passing through four points $(-1, 1)$, $(0, -1)$, $(1, 0)$, $(2, 2)$.
 - (a) Find a polynomial P whose graph passes through the given points. Make sure to simplify P .
 - (b) Use Matlab to plot the graph of P on the interval $[-2, 3]$.
 - (c) What is the position on the curve when $x = 1.5$? What is the slope at this point? In other words, find $P(1.5)$ and $P'(1.5)$.