Worksheet

1/29/2020

Name: ______________________________

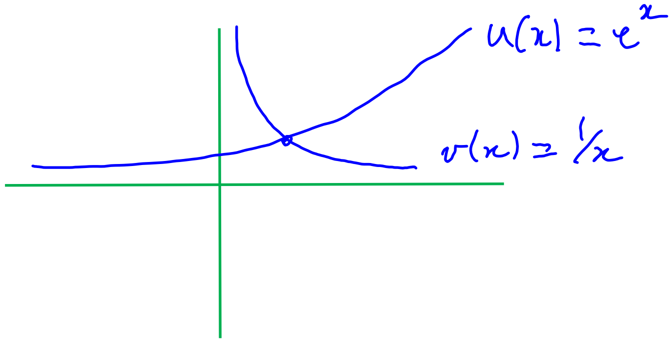
1. Newton's method can be used to compute approximately $\sqrt{3}$ by the procedure:
   - Find a function f such that $x_* = \sqrt{3}$ is a root.
   - Write the iteration formula of Newton's method.
   - Pick a point x_0 as the initial iteration. The closer x_0 is to x_* the better.
   - What do you get for x_4?

See Lecture 10

2. Find approximately the intersection point of the graph of $u(x) = e^x$ and the graph of $v(x) = 1/x$ by

   (a) Newton's method (3 iterations).

   (b) Bisection method (3 iterations).



The intersection point solves the equation

$$e^x = \frac{1}{x},$$

which is equivalent to $xe^x = 1$.

We will find approximate root of

$$f(x) = xe^x - 1.$$

(a) Use Newton's method:

We have $f'(x) = xe^x + e^x = (x+1)e^x$

Pick $x_0 = 1$. The iteration formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n e^{x_n} - 1}{(x_n+1)e^{x_n}}$$

$$= \frac{x_n^2 e^{x_n} + 1}{(x_n+1)e^{x_n}}$$

From here one can compute x_1, x_2, x_3.

$x_3 = \dots$ is the final answer.

(b) Observe that $f(1/2) = \frac{1}{2} e^{1/2} - 1 < 0$ and

$$f(1) = e - 1 > 0$$

One can pick the initial interval as

$$[a_0, b_0] = [\tfrac{1}{2}, 1]$$

Then find $[a_1, b_1]$, $[a_2, b_2]$, $[a_3, b_3]$.

$c_3 = \dfrac{a_3 + b_3}{2} = \ldots$ is the final answer.