KEY

Name ______________________________

M1151 – Fall 2012 – Final Exam – December 14, 2012

Signature ______________________________

Student ID ______________

Section Number ______________

For multiple choice questions (1-19) only your selected answer choice is corrected. For multiple choice, be sure to carefully select your answer on this test, and also on the bubble sheet. This will be explained carefully to you at the beginning of the exam.

For other questions (21-25), be sure to show all of your work and circle your final answer. If the question asks for an exact answer, that means do not use your calculator. Multiple choice questions are worth 10 points each, and the others are worth 25 points each. Total points possible are 340.

Problem	Possible points	Earned Points
Machine Graded 1-19	190	
20	25	
21	25	
22	25	
23	25	
24	25	
25	25	
26	25	
Total	340	

Grading

20 Lauren

21 Tuan

22 Richard

23 Scott

24 Duane

25 Guosheng

$\cos^2 x + \sin^2 x = 1$ $\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$ $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$\sin(2\theta) = 2\sin\theta\cos\theta$ $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$ $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

For the complex numbers $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$,

$$z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

For the complex number $= r(\cos\theta + i\sin\theta)$, $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

Ellipse with major axis parallel to x-axis; $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, a > b > 0, b^2 = a^2 - c^2$

Ellipse with major axis parallel to y-axis; $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, a > b > 0, b^2 = a^2 - c^2$

Hyperbola with transverse axis parallel to x-axis; $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, b^2 = c^2 - a^2$,
$(y-k) = \pm\frac{b}{a}(x-h)$

Hyperbola with transverse axis parallel to y-axis; $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1, b^2 = c^2 - a^2$,

$(y-k) = \pm\frac{a}{b}(x-h)$

Arithmetic sequence; $a_n = a_1 + (n-1)d$

Geometric sequence; $a_n = a_1 r^{n-1}$

Geometric series: $\sum_{k=1}^{n} a_1 r^{k-1} = \frac{a_1(1-r^n)}{1-r}$, *and if* $|r| < 1$, $\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1-r}$

Name ____

1) An object is traveling around a circle with a radius of 20 meters. If in 10 seconds a central angle of $\frac{1}{5}$ radian is swept out, what is the linear speed of the object?

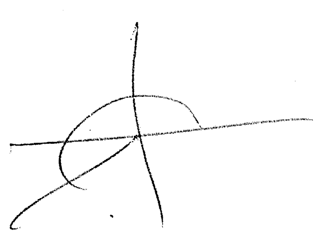
A) $\frac{1}{5}$ m/sec B) $\frac{2}{5}$ m/sec C) $\frac{1}{4}$ m/sec D) $\frac{1}{8}$ m/sec ✓

$v = \omega r$

$\omega = \frac{\frac{1}{5}}{10} \frac{R}{s} = \frac{1}{50} \frac{R}{s} \rightarrow v = \left(\frac{1}{50}\right)(20) = \frac{2}{5} \frac{m}{sec}$

2) The point (-2,-1) is on the terminal side an angle θ. Find the exact value of $\sec\theta$.

A) $-\frac{\sqrt{5}}{2}$ B) $-\frac{3\sqrt{5}}{5}$ C) $-\sqrt{5}$ D) $\frac{\sqrt{5}}{2}$ ✓



$\cos\theta = \frac{y}{r} = \frac{-2}{\sqrt{5}}$

$\rightarrow \sec\theta = \frac{-\sqrt{5}}{2}$

3) Find the exact value of cot(750°).

A) $-\sqrt{3}$ B) $\frac{\sqrt{3}}{3}$ C) $-\frac{\sqrt{3}}{3}$ D) $\sqrt{3}$

✓

$\tan(750°) = \tan(720° + 30°) = \tan(30°) = \frac{1}{\sqrt{3}}$

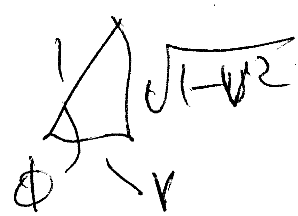
$\rightarrow \cot(750°) = \sqrt{3}$

4) Write the trigonometric expression $\cos(\sin^{-1} u + \cos^{-1} v)$ as an algebraic expression containing u and v.

A) $uv - (\sqrt{1-u^2})(\sqrt{1-v^2})$ C) $v\sqrt{1-u^2} + u\sqrt{1-v^2}$

B) $uv + (\sqrt{1-u^2})(\sqrt{1-v^2})$ D) $v\sqrt{1-u^2} - u\sqrt{1-v^2}$

✓



$\cos(\theta + \phi)$

$= \cos\theta \cos\phi - \sin\theta \sin\phi$

$= v\sqrt{1-u^2} - u\sqrt{1-v^2}$

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5) For the angle θ, $\sin\theta = \frac{1}{4}$, $0 < \theta < \frac{\pi}{2}$. Find $\sin\frac{\theta}{2}$.

A) $\frac{\sqrt{8-2\sqrt{15}}}{4}$ B) $\frac{\sqrt{8+2\sqrt{15}}}{4}$ C) $\frac{\sqrt{10}}{4}$ D) $\frac{\sqrt{6}}{4}$

✓

$$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{1}{16}} = \frac{\sqrt{15}}{4}$$

$$= \sqrt{\frac{1-\frac{\sqrt{15}}{4}}{2}}$$

$$= \sqrt{\frac{4-\sqrt{15}}{8}} = \sqrt{\frac{8-2\sqrt{15}}{16}} = \frac{\sqrt{8-2\sqrt{15}}}{4}$$

6) Find the exact value of $\cos^{-1}(-1)$.

A) 0 B) π C) 2π D) $\frac{\pi}{2}$

✓

7) Solve the triangle a=7, c=12, B=108⁰.

A) b=18.5, A=27⁰, C=45⁰
B) b=15.6, A=25⁰, C=47⁰
C) b=21.4, A=23⁰, C=49⁰
D) no triangle

✓

$b^2 = 12^2 + 7^2 - 2 \cdot 12 \cdot 7 \cos(108^0) \rightarrow b \approx 15.64$

$\frac{\sin 108^0}{15.64} \approx \frac{\sin C}{12} \rightarrow \sin C \approx \frac{12 \cdot \sin 108^0}{15.64}$

$\rightarrow C \approx 47^0$

8) Solve the triangle a=19, b=16, c=11.

A) A=57.3⁰, B=87.4⁰, C=35.3⁰
B) A=87.4⁰, B=35.3⁰, C=57.3⁰
C) A=87.4⁰, B=57.3⁰, C=35.3⁰
D) A=35.3⁰, B=57.3⁰, C=87.4⁰

✓

$\cos A = \frac{16^2 + 11^2 - 19^2}{2 \cdot 16 \cdot 11} \approx 1.52 \rightarrow A \approx 87.39^0$

$\sin B = \frac{16 \cdot \sin 87.4}{19} \approx .84$

$\rightarrow B \approx 57.27^0$

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9) Find the area of the triangle given by a=6, b=6, c=7.

A) 21.14 B) 18.25 C) 15.54 D) 17.06

$$\cos A = \frac{6^2+7^2-6^2}{2\cdot 6\cdot 7} \Rightarrow A \approx 54.31^\circ$$

$$\text{Area} \approx \tfrac{1}{2}(6)(7)\sin 54.31^\circ \approx 17.055$$

10) Form a polynomial f(x) with real coefficients having degree 4 and the zeros 1, -1, and 4-2i.

A) $f(x)=x^4 - 8x^3 + 19x^2 + 8x - 20$

B) $f(x)=x^4 - 8x^3 + 19x^2 + 8x + 20$

C) $f(x)=x^4 + 8x^3 + 19x^2 - 8x + 20$

D) $f(x)=x^4 + 8x^3 + 19x^2 - 8x - 20$

wrote on Testben

$$(x-1)(x+1)(x-(4+2i))(x-(4-2i))$$

$$= (x^2-1)(x^2-8x+20) = x^4-8x^3+19x^2+8x-20$$

11) The equation $r = 2(\sin\theta - \cos\theta)$ is given in polar coordinates. Write the equation using rectangular coordinates.

A) $2x^2 + 2y^2 = y - x$

B) $2x^2 + 2y^2 = x - y$

C) $x^2 + y^2 = 2y - 2x$

D) $x^2 + y^2 = 2x - 2y$

$\sqrt{x^2+y^2} = 2\left(\frac{y}{\sqrt{x^2+y^2}} - \frac{x}{x^2+y^2}\right)$

Changed from Totten

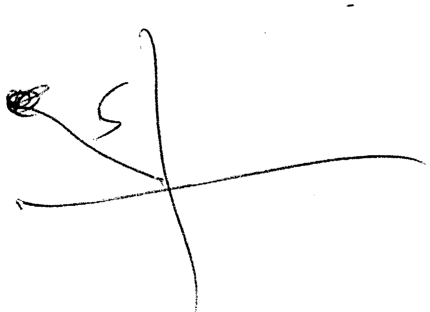
12) The polar coordinates of a point are $\left(5, \frac{-4\pi}{3}\right)$ Find the rectangular coordinates.

A) $\left(\frac{-5}{2}, \frac{5\sqrt{3}}{2}\right)$

B) $\left(\frac{-5\sqrt{3}}{2}, \frac{-5}{2}\right)$

C) $\left(\frac{5}{2}, \frac{-5\sqrt{3}}{2}\right)$

D) $\left(\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$



$x = 5\left(-\frac{1}{2}\right)$

$y = 5\left(\frac{\sqrt{3}}{2}\right)$

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13) Write the complex number 5i in polar form.

A) $5(\cos 90° + i \sin 90°)$ C) $5(\cos 0° + i \sin 0°)$

B) $5(\cos 180° + i \sin 180°)$ D) $5(\cos 270° + i \sin 270°)$

✓

14) Write the expression $(-\sqrt{3} + i)^6$ in standard form a + bi.

A) $-64\sqrt{3} + 64i$ C) $64i$

B) -64 D) $64 - 64i$

✓

$r = 2$

$\theta = \frac{5\pi}{6}$

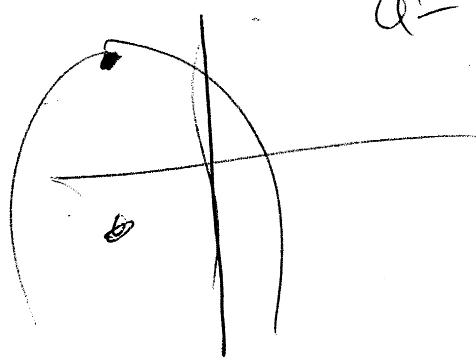
$2^6\left(\cos 5\pi + i \sin 5\pi\right)$

$= -64$

15) Find the vertex, focus, and directrix of the parabola $(x+3)^2 = -16(y-2)$.

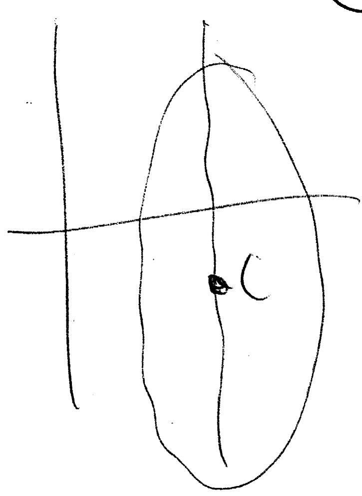
A) Vertex: (3,-2), Focus: (3,-6), Directrix: y=2
B) Vertex: (2,-3), Focus: (2,-7), Directrix: y=1
C) Vertex: (-3,2), Focus: (-3,-2), Directrix: y=6
D) Vertex: (-3,2), Focus: (-3,6), Directrix: x=-2

$a=4$



16) Find the center, foci, and vertices of the ellipse $36(x-3)^2 + 16(y+2)^2 = 576$.

A) Center: $(4,-2)$, Foci: $(4,-2-2\sqrt{5})$, $(4,-2+2\sqrt{5})$, Vertices: (4,4),(4,-8)
B) Center: $(-3,-2)$, Foci: $(-3,-2-2\sqrt{5})$, $(-3,-2+2\sqrt{5})$, Vertices: $(-3,4)$,(-3,-8)
C) Center: $(-2,3)$, Foci: $(-2,3-2\sqrt{5})$, $(-2,3+2\sqrt{5})$, Vertices: $(-2,4)$,(-2,-8)
D) Center: $(3,-2)$, Foci: $(3,-2-2\sqrt{5})$, $(3,-2+2\sqrt{5})$, Vertices: (3,4),(3,-8)



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$$\frac{(x-3)^2}{4^2} + \frac{(y+2)^2}{6^2} = 1$$

$a=4$
$b=2$
$c=2\sqrt{5}$

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17) Find the first term, the common difference, and give a recursive formula for the arithmetic sequence for which the 9th term is -41, and the 15th term is -77.

A) $a_1=13, d=6, a_n=a_{n-1}+6$

B) $a_1=7, d=6, a_n=a_{n-1}+6$

C) $a_1=7, d=-6, a_n=a_{n-1}-6$

D) $a_1=13, d=-6, a_n=a_{n-1}-6$

$$\begin{cases} a_9=-41=a_1+8d \\ a_{15}=-77=a_1+14d \end{cases}$$

$\Rightarrow$ $a_1=7$

$d=-6$

18) Find the nth term of the geometric sequence 2,6,18,54,162,...

A) $a_n = 2 \cdot 3n$ B) $a_n = 2 \cdot 3^{n-1}$ C) $a_n = 2 \cdot 3^n$ D) $a_n = a_1 + 3^n$

$r = 3$

$a_1 = 2$

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19) Determine whether the infinite geometric series $5 - \frac{5}{4} + \frac{5}{16} - \frac{5}{64} + \ldots$ converges or diverges. If it converges, find its sum.

A) Converges: $\frac{-5}{4}$

B) Converges: $\frac{15}{4}$

C) Converges: 4

D) Diverges

$r = -\frac{1}{4}$

$\left|-\frac{1}{4}\right| < 1$ so

Good

$\frac{5}{1-(-\frac{1}{4})} = \frac{5}{\frac{5}{4}} = 4$

20) A city is on the ocean and so has tides coming in. On December 14th, a high tide occurs at 8:00 PM. Assume that the high tides occur 21 hours apart. The height of the tide is measured by a marker on the beach. When the tide is the highest, it is 5 feet above the marker, and when the tide is the lowest, it is 1 foot below the marker. We want to write a cosine function which describes the height of the tide above or below the marker at time t.

8 points

A) What is the amplitude and the period of the trigonometric function which describes the motion?

$\text{Amp} = \frac{5-(-1)}{2} = 3$ 4pts

$\text{Period} = 21$ 4pts

8 points

B) Using t as the number of hours since 6:00 AM on December 14th, and y=f(t) as the number of feet above or below the marker, write a function which describes the height of the tide for any time t (starting at time t=0). The form of the function is $y = f(t) = A\cos(\omega t - \phi) + B$, or, $y = f(t) = A\cos\left(\omega(t - \frac{\phi}{\omega})\right) + B$.

Phase Shift = 14 (8PM – 6AM)

$\omega = \frac{2\pi}{T} = \frac{2\pi}{21}$

$B = \frac{5-1}{2} = 2$

$$\boxed{y = 3\cos\left(\frac{2\pi}{21}(t-14)\right) + 2 \quad \text{OR} \quad y = 3\cos\left(\frac{2\pi}{21}t - \frac{4\pi}{3}\right) + 2}$$

2pts 2pts 2pts 2pts

Name ____

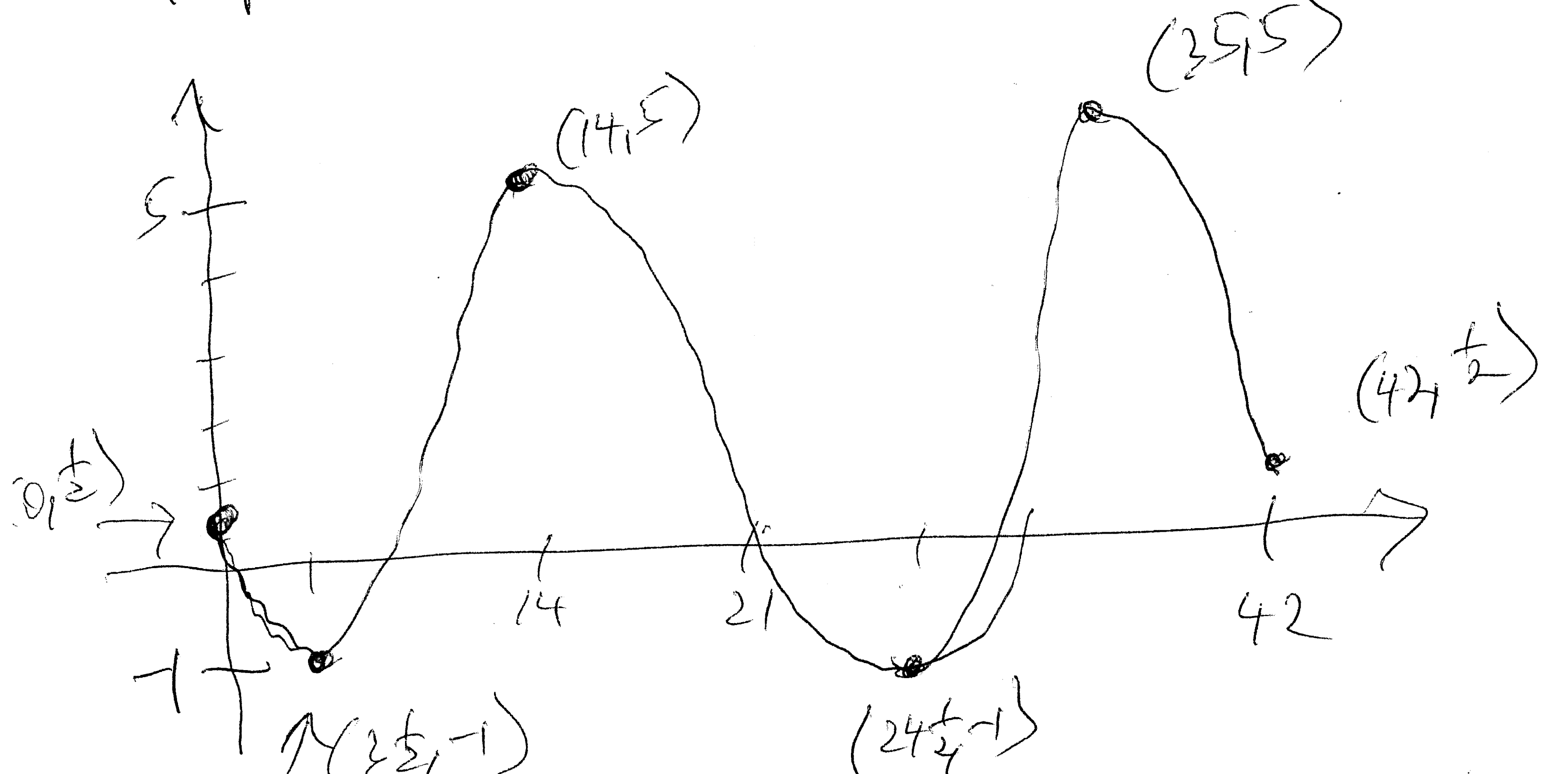
9 points

C) Graph the equation, showing **exactly** two periods of the function beginning with time t=0. Be sure to label the x and y values of key points with exact values, such as the beginning and end of each cycle, and the absolute minimum and absolute maximum values. You do not need to compute the exact values of the coordinates for the x-axis intercepts, but you do need to compute the exact value of the y-axis intercept (after all, that is where the first period starts).

$f(0) = 3\cos\left(-\frac{4\pi}{3}\right) + 2 = 3\left(-\frac{1}{2}\right) + 2 = \frac{1}{2}$

So start at $(0, \frac{1}{2})$ with Period = 21

& phase shift = 14



1 point for each of the six key points. 3 points at your discretion.

FOR #21 – If they show reasonable work but get wrong answer, give them work points (arith. error, for instance)

21) Solve the following systems of equations, remembering that solving a system of equation means there may be one solution, there may be no solutions, or there may be infinitely many solutions.

8 points A)

$$\begin{cases} x + 3y = 5 \\ 2x - 3y = -8 \end{cases}$$

Must show work.

4 points work
4 points

$\begin{bmatrix} 1 & 3 & 5 \\ 2 & -3 & -8 \end{bmatrix}$

$R_2 \leftarrow -2R_1 + R_2$

$\begin{bmatrix} 1 & 3 & 5 \\ 0 & -9 & -18 \end{bmatrix}$

$R_2 \leftarrow -\frac{1}{9}R_2$

$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \end{bmatrix} \rightarrow y = 2$

$\rightarrow x = 5 - 3y = -1$

$\boxed{(-1, 2)}$

B) 8 points

$$\begin{cases} x + 3y = 5 \\ 9y + 3x - 15 = 0 \end{cases}$$

Must show work.

4 points work
4 points answer

$\begin{bmatrix} 1 & 3 & 5 \\ 3 & 9 & 15 \end{bmatrix}$

$R_2 \leftarrow -3R_1 + R_2$

$\begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow x = 5 - 3y$

$\rightarrow \boxed{(5-3y, y) \text{ all real } y}$

OR any reasonable notation

OR

$3y = 5 - x$

$y = \frac{5-x}{3}$

$\boxed{\left(x, \frac{5-x}{3}\right) \text{ all real } x}$

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9 points

$$\begin{cases} x+y+z=6 \\ 2x-3=y+z \\ 2y+2z=-x \end{cases}$$

Must show work

5 points work
4 points answer

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & -1 & 3 \\ 1 & 2 & 2 & 0 \end{bmatrix}$$

$R_2 \Leftarrow -2R_1 + R_2$

$R_3 \Leftarrow -R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & -3 & -9 \\ 0 & 1 & 1 & -6 \end{bmatrix}$$

$R_2 \Leftarrow 3R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 0 & -27 \\ 0 & 1 & 1 & -6 \end{bmatrix} \longrightarrow \boxed{\text{No solution}}$$

OR

$\boxed{\text{inconsistent}}$

22) Exactly solve $2sin^2(3\theta) = 2\cos 3\theta + 2$ on the interval $0 \le \theta < 2\pi$ giving exact answers.

Using $\sin^2(3\theta) = 1 - \cos^2(3\theta)$

we get

$2(1 - \cos^2(3\theta)) - 2\cos(3\theta) - 2 = 0$

$\cancel{2} - 2\cos^2(3\theta) - 2\cos(3\theta) - \cancel{2} = 0$

$\cos^2(3\theta) + \cos(3\theta) = 0$

$\boxed{\cos(3\theta)(\cos(3\theta) + 1) = 0}$

$\boxed{\cos(3\theta) = 0}$

$3\theta = \frac{\pi}{2} + 2\pi k \to \theta = \frac{\pi}{6} + \frac{4\pi k}{6} \to \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6} = \frac{3\pi}{2}$

$3\theta = \frac{3\pi}{2} + 2\pi k \to \theta = \frac{3\pi}{6} + \frac{4\pi k}{6} \to \theta = \frac{3\pi}{6} = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\boxed{\cos(3\theta) = -1}$

$3\theta = \pi + 2\pi k \to \theta = \frac{\pi}{3} + \frac{2\pi k}{3}$

$\to \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

9 solutions. Give 2 points for each correct solution, and -2 for extraneous. 2 points for work.

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8 points

23) A parabola has vertex at (8,7) and the focus at (8,3).

A) Find an equation for the parabola.

$V=(8,7)$

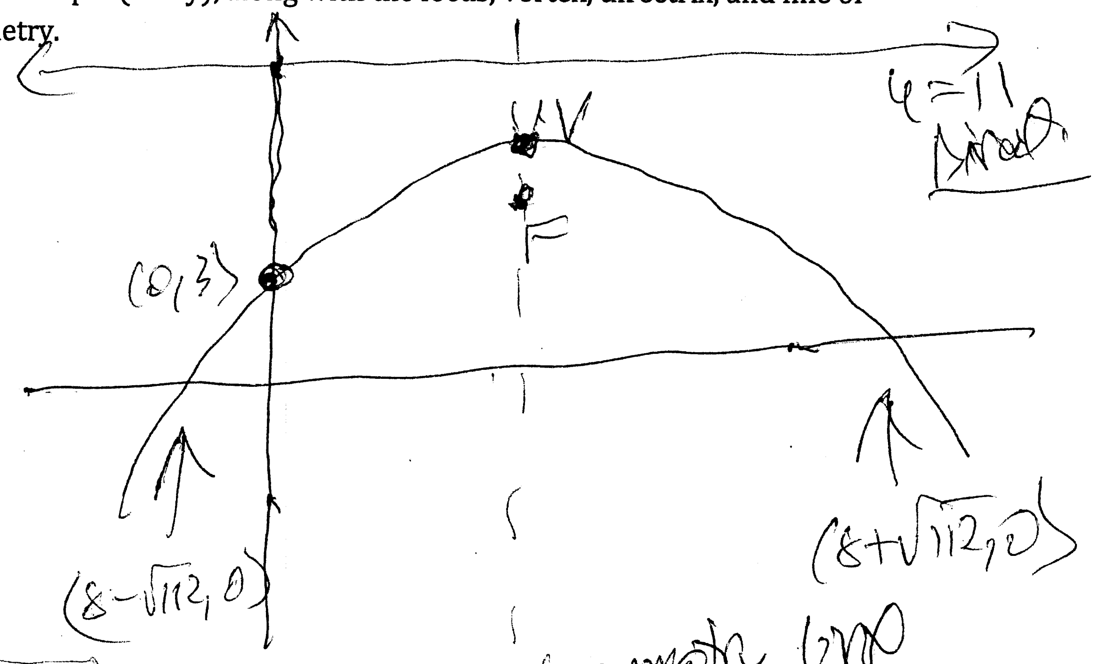
$F=(8,3) \Rightarrow a=4$

$$\boxed{-(y-7)16=(x-8)^2}$$

4 pts 2 pts 2 pts

17 points

B) Sketch the parabola, being sure to compute exactly and label the x and y-axis intercepts (if any), along with the focus, vertex, directrix, and line of symmetry.



$V=(8,7)$

$F=(8,3)$

Directrix: $y=11$

x-int $y=0$

$7\cdot 16=(x-8)^2$

$\pm\sqrt{112}=x-8$

$x=8\pm\sqrt{112}$

$\approx 8\pm 10.6 \approx (18.6, -2.6)$

y-int $x=0$

$-(y-7)16=64$

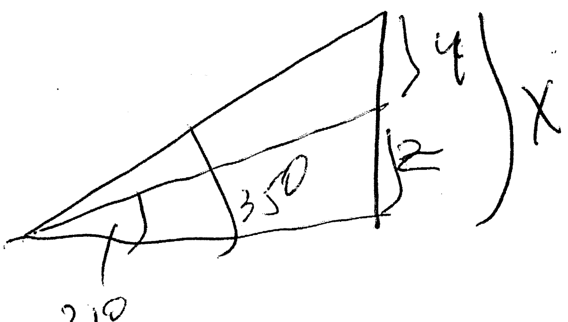
$16y=112-64=48$

$y=3$

2 pts for computing & labeling each of intercepts, focus, directrix, symmetry line & vertex (so 14 pts)

3 pts for matching the eqn in part A.

24) To measure the height of Lincoln's caricature on Mr. Rushmore, two sightings 800 feet from the base of the mountain are taken. If the angle of elevation to the bottom of Lincoln's face is 32^0, and the angle of elevation to the top of his face is 35^0, what is the height of Lincoln's face? Give your answer rounded to the nearest foot.



$\tan 35^0 = \frac{X}{800} \rightarrow X = 800 \tan 35^0$

≈ 560.160

$\tan 32^0 = \frac{z}{800} \rightarrow z = 800 \tan 32^0$

≈ 499.895

$y = X - z \approx 60.27 \approx 60$

12 points for reasonable work, even if leading to wrong conclusion, based on incorrect interpretation.

13 points for correct answer.

-3 points for incorrect rounding.

#25 5 pts for each correct value (ignore cosine since given)
No work points - little if any work need be shown here

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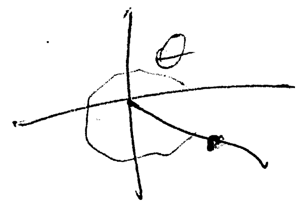
25) For an angle θ, $\cos\theta = \frac{2}{5}$, and $\tan\theta < 0$. Find the exact values of all six of the trigonometric functions for this angle.

$\cos\theta = \frac{2}{5}$ & $\tan\theta < 0 \Rightarrow$



$\sin\theta = -\sqrt{1-\left(\frac{2}{5}\right)^2} = -\sqrt{\frac{25-4}{25}} = \frac{-\sqrt{21}}{\sqrt{25}} = \frac{-\sqrt{21}}{5}$

$\tan\theta = \cancel{\left(\frac{2}{5}\right)} \frac{\left(\frac{-\sqrt{21}}{5}\right)}{\left(\frac{2}{5}\right)} = \frac{-\sqrt{21}}{5}\cdot\frac{5}{2} = -\frac{\sqrt{21}}{2}$

$\cot\theta = \frac{-2}{\sqrt{21}}$ $\quad \sec\theta = \frac{5}{2}$ $\quad \csc\theta = \frac{-5}{\sqrt{21}}$

$\sin\theta = \frac{-\sqrt{21}}{5}$ $\quad \tan\theta = \frac{-\sqrt{21}}{2}$ $\quad \cancel{\cot\theta =}$

$\csc\theta = \frac{-5}{\sqrt{21}} = \frac{-5\sqrt{21}}{21}$ $\quad \cot\theta = \frac{-2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$

$\sec\theta = \frac{5}{2}$

either answer
(no penalty for not rationalizing denominator)