White

1) $f(x) = -5x - 3$, $g(x) = 5x + 3$.

$(f \circ g)(x) = f(g(x)) = f(\underbrace{5x+3}_{u}) = -5u - 3$

Substitute back: $-5u - 3 = -5(5x+3) - 3 = -25x - 18$

A

2) * Find $\sin(\sin^{-1} \frac{1}{7})$:

Put $\theta = \sin^{-1} 1/7$. Then $-\pi/2 \leq \theta \leq \pi/2$ and $\sin\theta = 1/7$.

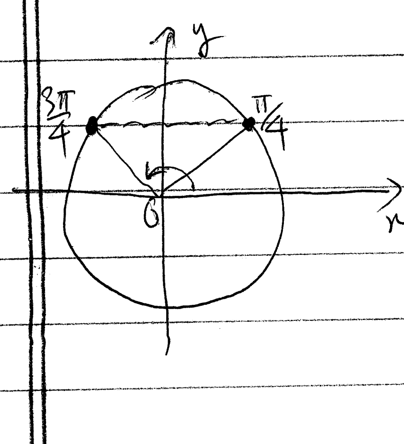
$\text{Answer} = \frac{1}{7}$

* Find $\sin^{-1}(\sin(\frac{3\pi}{4}))$

, Put $\theta = \sin^{-1}(\sin \frac{3\pi}{4})$

, Then $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $\sin\theta = \sin \frac{3\pi}{4}$

, Cannot choose $\theta = 3\pi/4$ because it's out of the interval.



, Take reflection w.r.t y-axis

$\theta = \frac{\pi}{4}$

C

3) $\sin(\tan^{-1} u)$

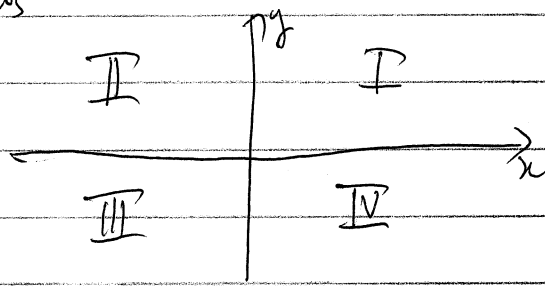
- Put $\theta = \tan^{-1} u$. Then $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and $\tan\theta = u$

- Need to find $\sin\theta$

- Note that $\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{1}{1+\tan^2\theta} = 1 - \frac{1}{1+u^2} = \frac{u^2}{1+u^2}$

- Thus $\sin\theta = \pm \frac{u}{\sqrt{1+u^2}}$

- Choose the plus sign because $\cos\theta > 0$ in the 1^{st} and 4^{th} quadrants

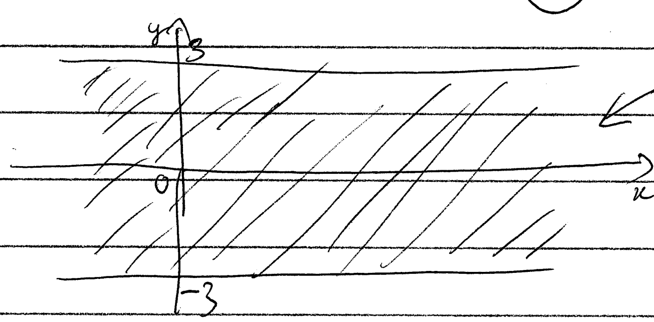


- Therefore, $\sin\theta = \frac{u}{\sqrt{1+u^2}} = \frac{u\sqrt{u^2+1}}{u^2+1}$ D

4) $f(x) = -3\cos(10x+9)$

- Amplitude $= 3$. Therefore the range of f is $[-3,3]$
- The range of f is the domain of f^{-1}
- Therefore the domain of f^{-1} is $[-3,3]$

B



graph of f oscilates in this strip.

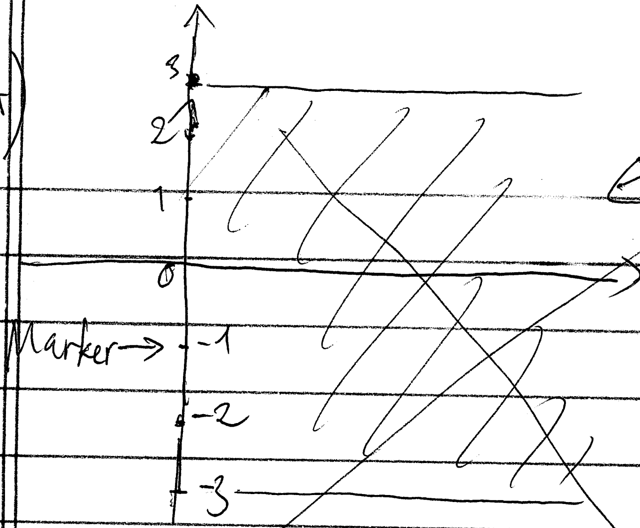
6) $f(x) = \sin x$

$h(x) = \tan x$

Find $f \circ h^{-1}\left(-\frac{7}{24}\right)$

- $f\left(h^{-1}\left(-\frac{7}{24}\right)\right) = \sin\left(\tan^{-1}\left(-\frac{7}{24}\right)\right)$
- Put $\theta = \tan^{-1}\left(-\frac{7}{24}\right)$
- Then $-\pi/2 < \theta < \pi/2$ and $\tan\theta = -7/24$
- Need to find $\sin\theta$
- we have $\cos^2\theta = \dfrac{1}{1+\tan^2\theta} = \dfrac{1}{1+\left(-7/24\right)^2} = \dfrac{24^2}{25^2}$
- Thus $\cos\theta = \pm\dfrac{24}{25}$
- Because $-\pi/2 < \theta < \pi/2$ implies $\cos\theta > 0$, we choose $\cos\theta = \dfrac{24}{25}$
- $\sin\theta = (\tan\theta)(\cos\theta) = -\dfrac{7}{24}\,\dfrac{24}{25} = -\dfrac{7}{25}$

7) A)

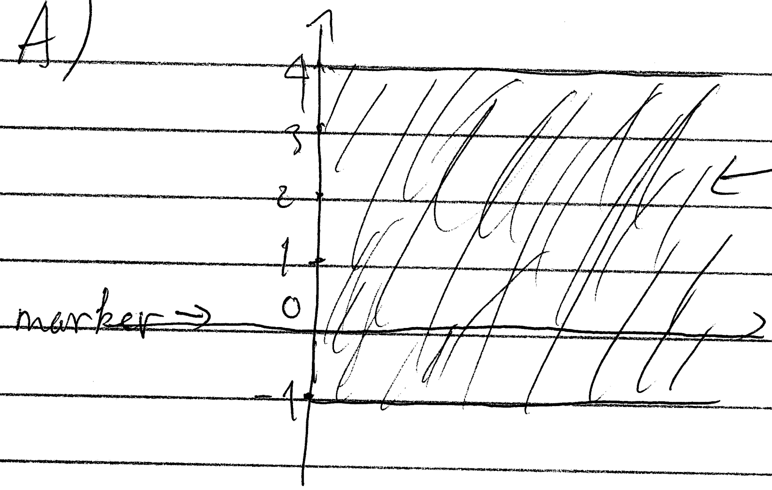


the function oscilates in this strip.

Amplitude = 3 $\left(= \frac{max - min}{2} = \frac{4-(-2)}{2} = 3 \right)$

B)

7) A)



$$\text{Amplitude} = \frac{\max - \min}{2} = \frac{4-(-1)}{2} = \frac{5}{2}$$

Period = 18 (hours)

B) $\omega = \frac{2\pi}{T} = \frac{2\pi}{18} = \frac{\pi}{9}$

Phase-shift $= 9 - 6 = 3$

$$\text{vertical shift} = \frac{\max + \min}{2} = \frac{4 + (-1)}{2} = \frac{3}{2}$$

$$y(t) = \frac{5}{2}\cos\left(\frac{\pi}{9}(t-3)\right) + \frac{3}{2}$$

or

$$y(t) = \frac{5}{2}\cos\left(\frac{\pi}{9}t - \frac{\pi}{3}\right) + \frac{3}{2}$$

c) · Two periods $= 2 \times 18 = 36$ (hours)

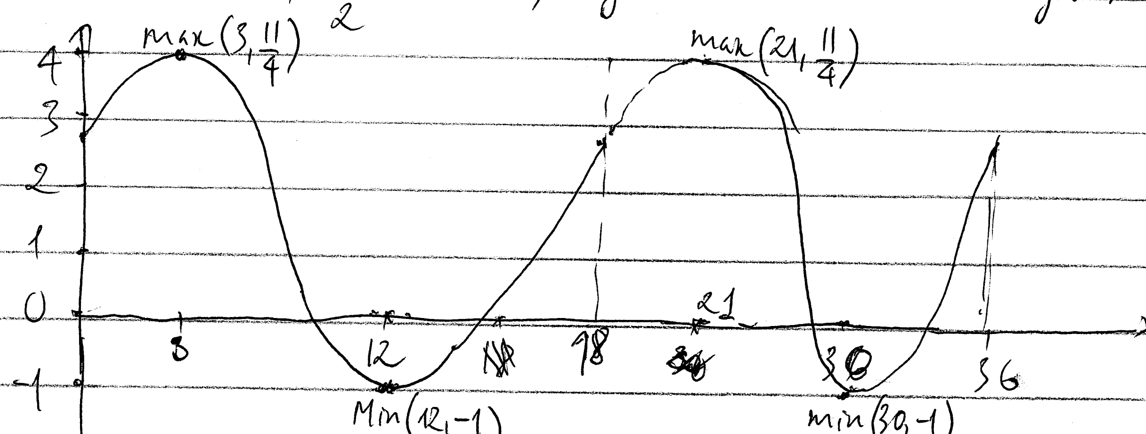
· It's enough to draw ~~one~~ one period, then just copy and paste.

· At $t = 0$, $y(0) = \frac{5}{2}\cos\left(-\frac{\pi}{3}\right) + \frac{3}{2} = \frac{5}{2}\cos\frac{\pi}{3} + \frac{3}{2}$

$$= \frac{5}{2}\,\frac{1}{2} + \frac{3}{2} = \frac{11}{4}$$

· When $t = 3$, $y(3) = \frac{5}{2}\cos 0 + \frac{3}{2} = \frac{5}{2} + \frac{3}{2} = 4$

· when $t = 3 + \frac{18}{2} = 12$, y is at minimum $y(12) = -1$



8) $\dfrac{\cot^2\theta}{\csc\theta - 1} = \dfrac{1+\sin\theta}{\sin\theta}$

1 Start from the left hand side. Convert everything to sine and Cosine

$$\frac{\cot^2\theta}{\csc\theta-1} = \frac{\left(\frac{\cos}{\sin}\right)^2}{\frac{1}{\sin}-1} = \frac{\frac{\cos^2}{\sin^2}}{\frac{1-\sin}{\sin}} = \frac{\cos^2}{\sin^2}\,\frac{\sin}{1-\sin}$$

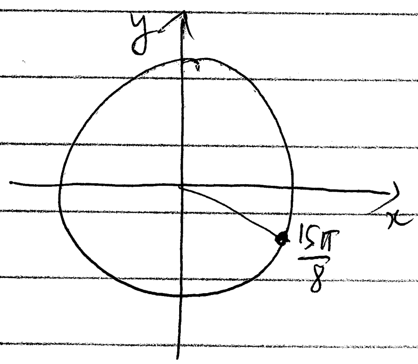
$$= \frac{\cos^2}{\sin(1-\sin)} = \frac{1-\sin^2}{\sin(1-\sin)} = \frac{(1+\sin)\cancel{(1-\sin)}}{\sin\cancel{(1-\sin)}}$$

$$= \frac{1+\sin}{\sin}$$ = the right hand side

9) - Find $\cos\left(\frac{15\pi}{8}\right)$ and $\sin\left(\frac{23\pi}{12}\right)$

- We'll use the half-angle identity because we know $\frac{15\pi}{4}$ and $\frac{23\pi}{6}$ better than $\frac{15\pi}{8}$ and $\frac{23\pi}{8}$

- $\cos\left(\frac{15\pi}{8}\right) = \pm\sqrt{\frac{1+\cos\left(\frac{15\pi}{4}\right)}{2}}$



- we choose the plus sign

- $\cos\left(\frac{15\pi}{8}\right) = \cos\left(4\pi - \frac{\pi}{4}\right)$

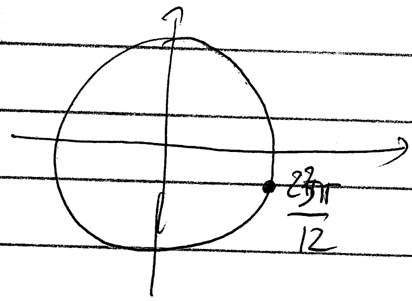
$= \cos\left(-\frac{\pi}{4}\right)$

$= \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

- Therefore, $\cos\frac{15\pi}{8} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$

$= \sqrt{\frac{2+\sqrt{2}}{4}}$

$$\sin\left(\frac{23\pi}{12}\right) = \pm\sqrt{\frac{1-\cos\frac{23\pi}{6}}{2}}$$



choose the minus sign

$$\cos\frac{23\pi}{6} = \cos\left(4\pi - \frac{\pi}{6}\right)$$

$$= \cos\left(-\frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

Therefore, $\sin\left(\frac{23\pi}{12}\right) = -\sqrt{\frac{1-\sqrt{3}/2}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}}$

10) A) $2\cos^2\theta - 3\cos\theta + 1 = 0, \quad 0 \leq \theta < 2\pi$

- Put $t = \cos\theta$
- $2t^2 - 3t + 1 = 0$
- Factoring the left hand side gives $(2t-1)(t-1) = 0$
- Then $t = 1/2$ or $t = 1$
- Then $\cos\theta = \frac{1}{2}$ or $\cos\theta = 1$
- We see that $\cos\frac{\pi}{3} = \frac{1}{2}$ and $\cos 0 = 1$
- We get $\theta = \frac{\pi}{3} + 2k\pi$ or $\theta = 0 + 2k\pi = 2k\pi$

  $\theta = -\frac{\pi}{3} + 2k\pi$ $\quad\quad$ $\theta = -0 + 2k\pi = 2k\pi$
- Only choose $\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}$

  because $0 \leq \theta < 2\pi$

B) $1+\cos\theta = 2\sin^2\theta, \quad 0 \leq \theta < 2\pi$

- We try to put everything in terms of cosine
- See that $\sin^2\theta = 1-\cos^2\theta$
- $1+\cos\theta = 2(1-\cos^2\theta)$
- Put $t = \cos\theta$
- $1+t = 2(1-t^2)$
- $2t^2+t-1=0$
- Factoring the left hand side $(t+1)(2t-1)=0$
- Get $t=-1$ or $t=\frac{1}{2}$
- $\cos\theta=-1$ or $\cos\theta=\frac{1}{2}$
- See that $\cos\pi=-1$ and $\cos\frac{\pi}{3}=\frac{1}{2}$
- Thus $\theta=\pi+2k\pi$ or $\theta=\frac{\pi}{3}+2k\pi$

  $\theta=-\pi+2k\pi$ $\qquad$ $\theta=-\frac{\pi}{3}+2k\pi$
- Only choose $\theta=\frac{\pi}{3}, \pi, \frac{5\pi}{3}$ because $0\leq\theta<2\pi$