White

1) $f(x) = -5x \cancel{+} -3$, $g(x) = 5x+3$

$(g \circ f)(x) = g(f(x)) = g(\underbrace{-5x \cancel{+} 3}_{u}) = 5u+3$

Substituting back : $5u+3 = 5(-5x-3)+3 = -25x-12$

D

2) $\sin\left(\sin^{-1}\frac{1}{7}\right)$ and $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$

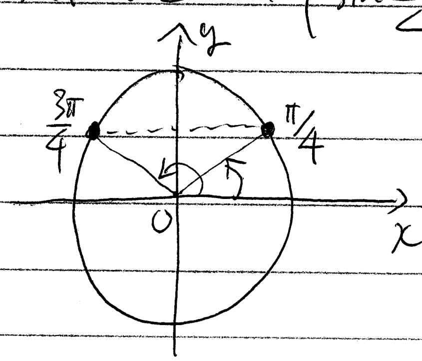
* Find $\sin\left(\sin^{-1}\frac{1}{7}\right)$:

Put $\theta = \sin^{-1}\frac{1}{7}$. Then $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $\sin\theta = \frac{1}{7}$

$\boxed{\text{Answer} = \frac{1}{7}}$

* Find $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$:

Put $\theta = \sin^{-1}\left(\sin\frac{3\pi}{4}\right)$. Then $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $\sin\theta = \sin\frac{3\pi}{4}$



$\frac{3\pi}{4}$ is <u>not</u> in between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

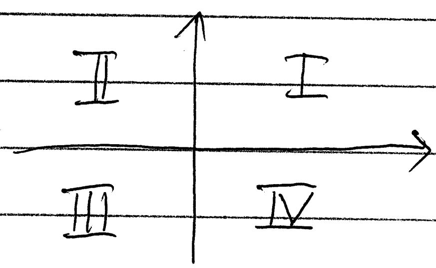
$\Rightarrow$ take reflection w.r.t y-axis

$\boxed{\theta = \frac{\pi}{4}}$

C

3) $\cos(\tan^{-1} u)$

- Put $\theta = \tan^{-1} u$. Then $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and $\tan\theta = u$.
- Need to find $\cos\theta$
- Note that $\cos^2\theta = \dfrac{1}{1+\tan^2\theta} = \dfrac{1}{1+u^2}$
- Because $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, it's in the quadrant I or IV



Thus $\cos\theta \geqslant 0$

- Therefore

$$\cos\theta = \sqrt{\frac{1}{1+u^2}} = \frac{1}{\sqrt{1+u^2}} = \frac{\sqrt{1+u^2}}{1+u^2}$$

C

4) $f(x) = -2\sin(5x + 19)$

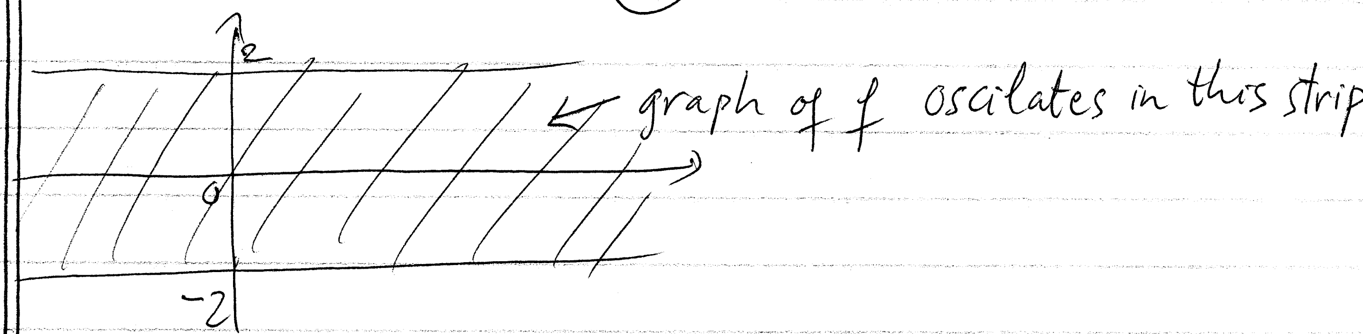
∵ Amplitude $= 2$. Therefore the range of f is

$[-2, 2]$.

∵ The range of f is the domain of f^{-1}

∴ Therefore the domain of f^{-1} is $[-2, 2]$

C

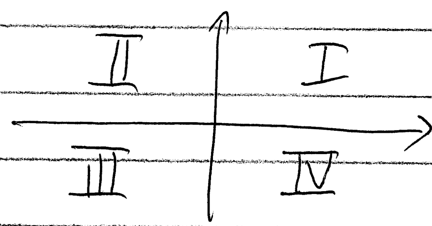


6) $g(x) = \cos x$

$h(x) = \tan x$

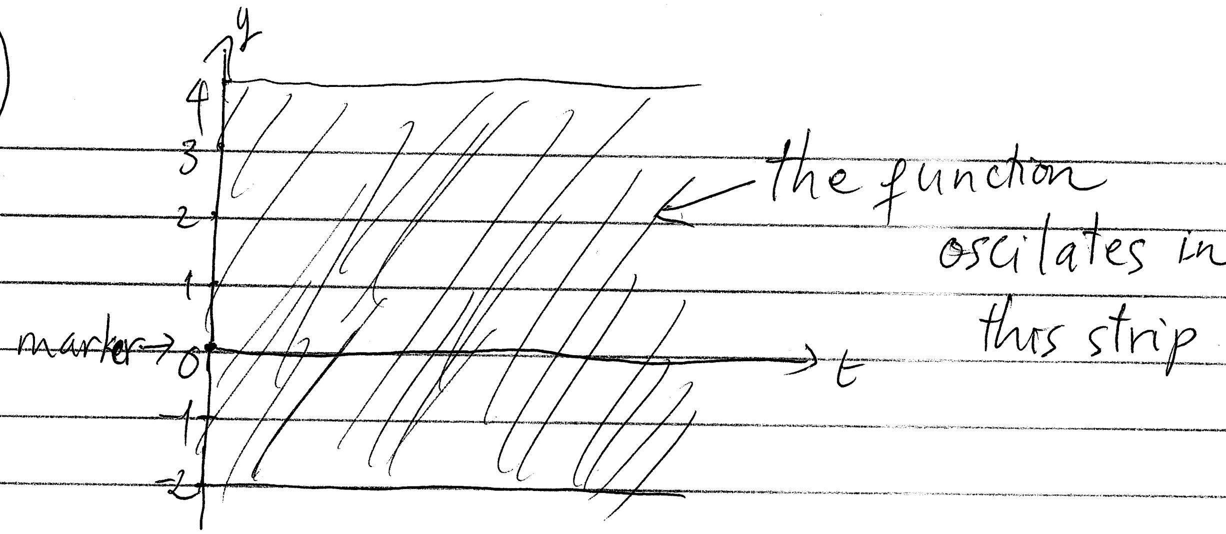
Find $(g\circ h^{-1})\left(-\frac{4}{5}\right)$

- ~~Find~~ $g\circ h^{-1}\left(\frac{4}{5}\right) = g\left(h^{-1}\left(-\frac{4}{3}\right)\right) = \cos\left(\tan^{-1}\left(-\frac{4}{5}\right)\right)$
- Put $\theta = \tan^{-1}\left(-\frac{4}{5}\right)$
- Then $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and $\tan\theta = -\frac{4}{5}$
- Need to find $\cos\theta$
- We have $\cos^2\theta = \frac{1}{1+\tan^2\theta} = \frac{1}{1+\left(-\frac{4}{5}\right)^2} = \frac{25}{41}$
- Because $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, it's in the 1^{st} or 4^{th} quadrant



- Thus $\cos\theta \geq 0$
- Thus $\cos\theta = \sqrt{\frac{25}{41}} = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$

7) A)



$$\text{Amplitude} = \frac{\max - \min}{2} = \frac{4-(-2)}{2} = 3$$

$$\text{Period} = 30 \text{(hours)}$$

B)

$$\omega = \frac{2\pi}{\text{period}} = \frac{2\pi}{30} = \frac{\pi}{15}$$

$$\text{Phase shift} = 11 - 6 = 5$$

$$\text{Vertical shift} = 1$$

$$y(t) = A\cos\left(\omega\left(t - \frac{\phi}{\omega}\right)\right) + B$$

$$= 3\cos\left(\frac{\pi}{15}(t-5)\right) + 1$$

Or $$y(t) = 3\cos\left(\frac{\pi}{15}t - \frac{\pi}{3}\right) + 1$$

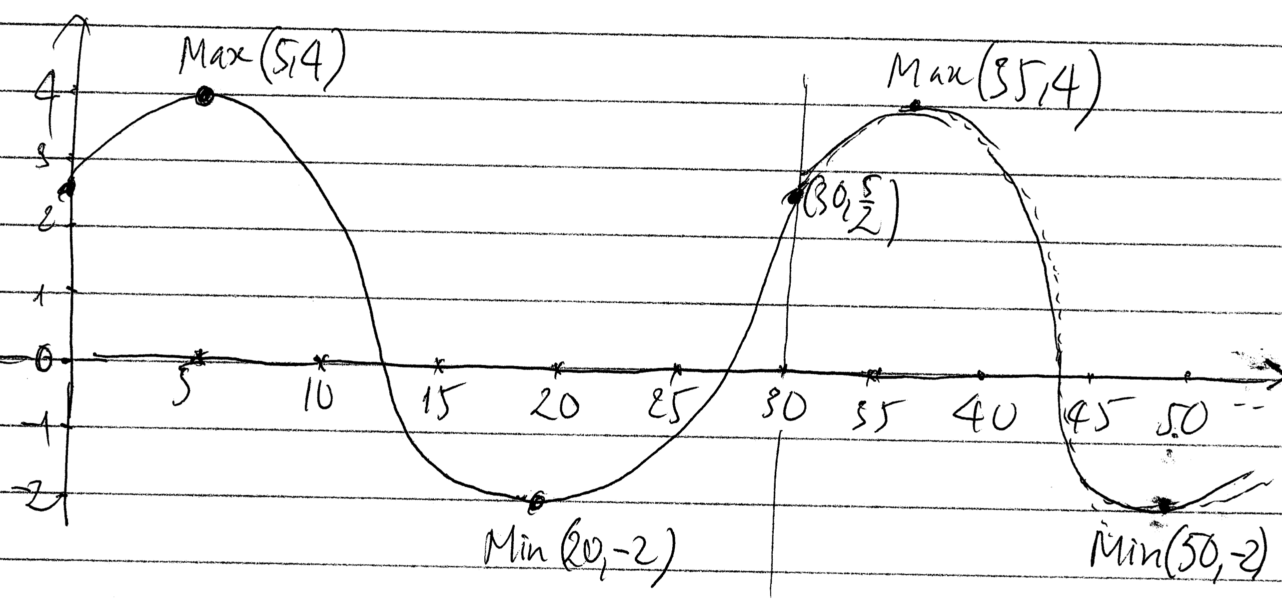
c) - Two periods $= 2\times 30 = 60$ (hours)

- It's enough to draw on one period, then just copy and paste.

- At $t=0$, $y(0) = 3\cos\left(-\frac{\pi}{3}\right)+1 = 3\cos\frac{\pi}{3}+1 = \frac{5}{2}$

- At $t=5$, we already know the tide is at peak i.e. $y(5)=4$

- At $t = 5+\frac{30}{2} = 20$, y is at minimum $y(20)=-2$



8) $$\frac{\cot^2\theta}{\csc\theta + 1} = \frac{1-\sin\theta}{\sin\theta}$$

Start from the left hand side. Convert everything to sine and cosine

$$\frac{\cot^2\theta}{\csc\theta + 1} = \frac{\left(\frac{\cos}{\sin}\right)^2}{\frac{1}{\sin} + 1} = \frac{\frac{\cos^2}{\sin^2}}{\frac{1+\sin}{\sin}} = \frac{\cos^2}{\sin^2} \cdot \frac{\sin}{1+\sin}$$

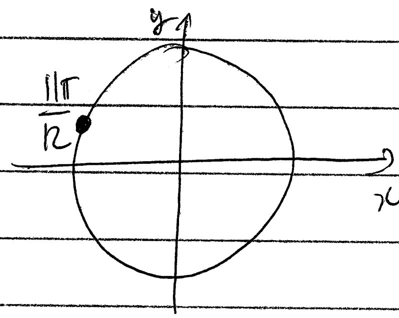
$$= \frac{\cos^2}{\sin(1+\sin)} = \frac{1-\sin^2}{\sin(1+\sin)} = \frac{(1-\sin)\cancel{(1+\sin)}}{\sin\cancel{(1+\sin)}}$$

$$= \frac{1-\sin}{\sin}$$ = the right hand side

9) $\cos\left(\frac{11\pi}{12}\right)$ and $\sin\left(\frac{7\pi}{8}\right)$

- We'll use the half-angle identity because we know $\frac{11\pi}{6}$ and $\cancel{11\pi}\ \frac{7\pi}{4}$ better than $\frac{11\pi}{12}$ and $\frac{7\pi}{8}$

- $$\cos\left(\frac{11\pi}{12}\right) = \pm\sqrt{\frac{1+\cos\left(\frac{11\pi}{6}\right)}{2}}$$

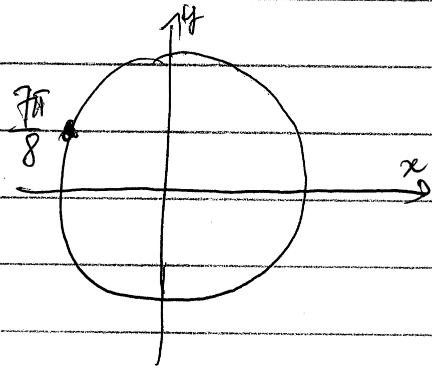


- We choose the minus sign

- $$\cos\left(\frac{11\pi}{6}\right) = \cos\left(2\pi - \frac{\pi}{6}\right)$$
$$= \cos\left(-\frac{\pi}{6}\right)$$
$$= \cos\left(\frac{\pi}{6}\right)$$
$$= \frac{\sqrt{3}}{2}$$

Therefore $$\cos\left(\frac{11\pi}{12}\right) = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$\sin\left(\frac{7\pi}{8}\right) = \sqrt{\frac{1-\cos\left(\frac{7\pi}{4}\right)}{2}}$$



choose the plus sign

$$\cos\left(\frac{7\pi}{4}\right) = \cos\left(2\pi - \frac{\pi}{4}\right)$$
$$= \cos\left(-\frac{\pi}{4}\right)$$
$$= \cos\frac{\pi}{4}$$
$$= \frac{\sqrt{2}}{2}$$

Therefore, $\sin\left(\frac{7\pi}{8}\right) = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$

10. A) $2\sin^2\theta - 3\sin\theta - 2 = 0, \quad 0 \le \theta < 2\pi$

- Put $t = \sin\theta$
- $2t^2 - 3t - 2 = 0$
- Factoring the left hand side gives $(2t+1)(t-2) = 0$
- Then $t = -\frac{1}{2}$ or $t = 2$
- Then $\sin\theta = -\frac{1}{2}$ or $\underbrace{\sin\theta = 2}_{\text{eliminate}}$
- $\sin\theta = -\frac{1}{2}$
- Find an angle θ_0 satisfying $\sin\theta_0 = -\frac{1}{2}$; $\theta_0 = -\frac{\pi}{6}$
- Then $\theta = -\frac{\pi}{6} + 2k\pi$ or $\theta = \pi - \left(-\frac{\pi}{6}\right) + 2k\pi$
- ~~Or~~ Then $\theta = -\frac{\pi}{6} + 2k\pi$ or $\theta = \frac{7\pi}{6} + 2k\pi$
- Only choose $\theta = \frac{2\pi}{6}, \frac{11\pi}{6}$

  because $0 \le \theta < 2\pi$

B) $\sin^2\theta - \cos^2\theta + \cos\theta = 0$, $0 \leq \theta < 2\pi$

- We try to put everything into terms of cosine
- See that $\sin^2\theta = 1 - \cos^2\theta$
- $(1-\cos^2\theta) - \cos^2\theta + \cos\theta = 0$
- Put $t = \cos\theta$
- $(1-t^2) - t^2 + t = 0$
- $-2t^2 + t + 1 = 0$
- Factoring, $(-t+1)(2t+1) = 0$
- Get $t = 1$ and $t = -\frac{1}{2}$
- $\cos\theta = 1$ and $\cos\theta = -\frac{1}{2}$
- See that $\cos 0 = 1$ and $\cos\frac{2\pi}{3} = -\frac{1}{2}$
- Thus $\theta = 0 + k2\pi$ and $\theta = \frac{2\pi}{3} + k2\pi$

  $\theta = -0 + k2\pi$ $\quad$ $\theta = -\frac{2\pi}{3} + k2\pi$
- Only choose $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ because $0 \leq \theta < 2\pi$