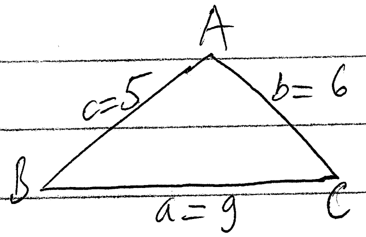
1) This is exactly Question 4 in Quiz 5, which you can find on the webpage. The answer is $A = 35^\circ.94$; $C \approx 95.06^\circ$ and $c = 11.88$.

2) This is exactly Question 4 in Quiz 6, which you can find on the webpage. The answer is $x^2 + y^2 = x$.

3)



Here we're given 3 sides. Thus we can use the law of cosine directly.

We have

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \approx \frac{6^2 + 5^2 - 9^2}{2.6.5} \approx -0.3333$$

Thus $A = \cos^{-1}(-0.3333) \approx 109.47^\circ$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{5^2 + 9^2 - 6^2}{2.5.9} \approx 0.7777$$

Thus $B = \cos^{-1}(0.7777) \approx 38.94^\circ$

$$C = 180^\circ - A - B = \cancel{109.47}\ 180^\circ - 109.47^\circ - 38.94^\circ \approx 31.6^\circ$$

So the best anwer among those given is $A = 109.5^\circ$, $B = 38.9^\circ$; $C = 31.6^\circ$

4) $f(x) = 7x^4 - x^2 + 2$

- A potential rational zeros of $f(x)$ is of the form p/q where
  - p divides 2, the constant term,
  - q divides 7, the leading coefficient.

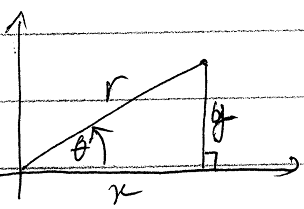
- Therefore $p \in \{\pm 1, \pm 2\}$ and $q \in \{1, 7\}$. Note that we

can <u>always</u> assume that q is positive because the minus sign

of q can be absorbed into p.

- Then $\frac{p}{q} \in \left\{ \pm 1, \pm 2, \pm \frac{1}{7}, \pm \frac{2}{7} \right\}$

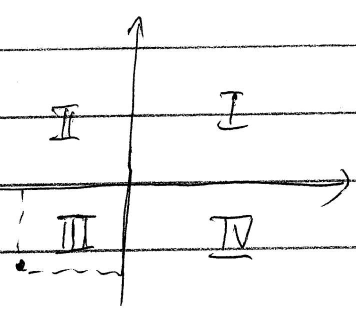
5) Remember that, to convert (x,y)-coordinates into (r,θ)-coordinate, we use the following formula $\begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \dfrac{y}{x} \end{cases}$



We're given $(x,y) = (-\sqrt{3}, -1)$. Thus

$$\begin{cases} r = \sqrt{x^2+y^2} = \sqrt{3+1} = 2 \\ \tan\theta = \dfrac{y}{x} = \dfrac{-1}{-\sqrt{3}} = \dfrac{1}{\sqrt{3}} \end{cases}$$

Now θ can be $\frac{\pi}{6}$ or $\frac{\pi}{6} + \pi = \frac{7\pi}{6}$ and we have to decide which to choose. Remember that the angle $\frac{\pi}{6}$ is in the quadrant I



while the angle $\frac{7\pi}{6}$ is in quadrant IV.

Because the given point $(x,y) = (-\sqrt{3}, -1)$, we is in quadrant III, we choose $\theta = \frac{7\pi}{6}$.

Thus, one polar coordinate of the given point is $(r,\theta) = \left(2, \frac{7\pi}{6}\right)$

We can add $2k\pi$ to θ to obtain various polar coordinates of the point: $(r,\theta) = \left(2, \frac{7\pi}{6} + 2k\pi\right)$

With $k = -1$, we get $(r,\theta) = \left(2, -\frac{5\pi}{6}\right)$. Thus we choose C.

6) $f(x) = x^4 - 3x^3 + 5x^2 - x - 10$

A) The possible rational zeros of $f(x)$ are of the form $\frac{p}{q}$ where

- p is a divisor of -10. $\rightarrow p \in \{\pm 1, \pm 2, \pm 5, \pm 10\}$

- q is a divisor (positive) of $1 \rightarrow q = 1$

Therefore all possible rational zeros of $f(x)$ are $\{\pm 1, \pm 2, \pm 5, \pm 10\}$

B) We'll find all real zeros of $f(x)$ by first finding the rational zeros and then factor $f(x)$.

- We test $\pm 1, \pm 2, \pm 5, \pm 10$ for zeros of $f(x)$ by means of long division:

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & 5 & -1 & -10 \\ & & -1 & 4 & -9 & 10 \\ \hline & 1 & -4 & 9 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} +2 & 1 & -4 & 9 & -10 \\ & & +2 & -4 & 10 \\ \hline & 1 & -2 & 5 & 0 \end{array}$$

We see that -1 and 2 are zeros of $f(x)$.

- Factor $f(x)$. We have $f(x) = (x+1)(x-2)(x^2-2x+5)$ by looking at the long division above.

- The polynomial $x^2-2x+5=0$ has no real zero because

$\Delta' = 1^2 - 5 = -4 < 0.$

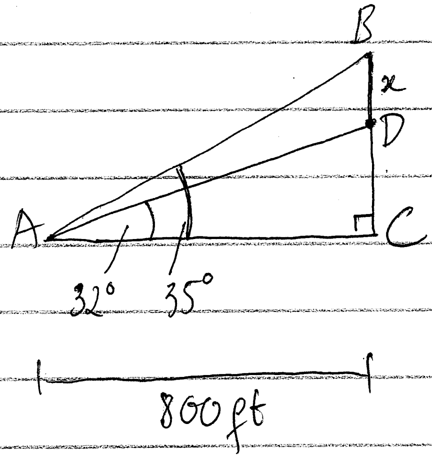
- Therefore, all real zeros of $f(x)$ are -1 and 2.

C) As we see above, the factorization of $f(x)$ over the real system is

$$f(x) = (x+1)(x-2)(x^2-2x+5)$$

The polynomial x^2-2x+5 couldn't be factored over the real system because it has no real zero.

7) This is problem 71, section 8.1, page 515 in the textbook.



- In the right triangle ABC, we have

$$BC = AC\tan 35^\circ$$
$$= 800\tan 35^\circ$$

- In the right triangle ADC, we have

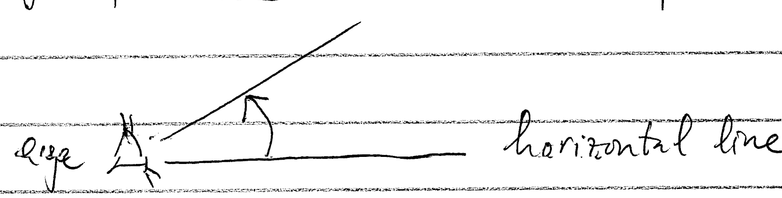
$$CD = AC\tan 32^\circ$$
$$= 800\tan 32^\circ$$

- Therefore,

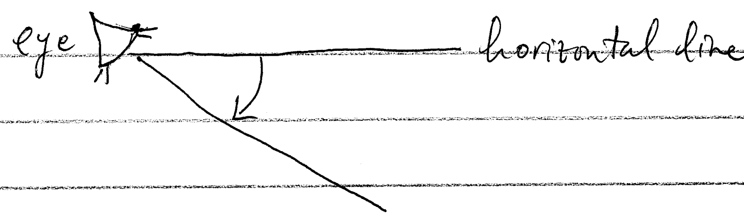
$$x = BC - CD = 800\tan 35^\circ - 800\tan 32^\circ$$
$$= 800(\tan 35^\circ - \tan 32^\circ)$$
$$\approx 60.27 \text{ ft}$$

- We round the answer to the nearest foot: $x \approx 60$ ft.

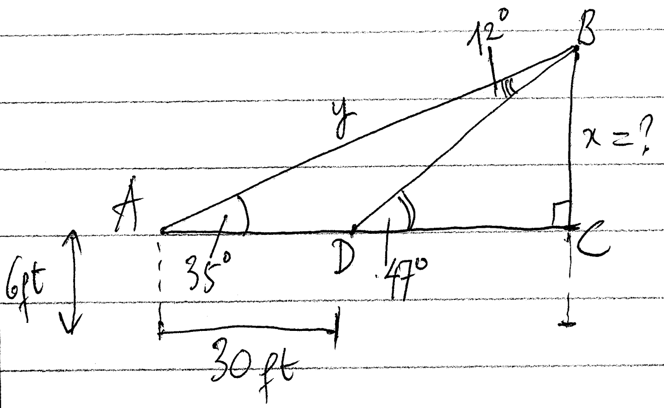
<u>Note</u> - The angle of <u>elevation</u> measures how "up" our sight is.



- The angle of <u>depression</u> measures how "down" our sight is.



8) The first step is to draw a correct picture.



In order to compute x, we can start with computing the hypotenuse $y = AB$ in the picture.

In the triangle ABD, we know $AD = 30ft$. The angle $\angle ADB$ is given by $180° - 47° = 133°$. In this triangle, $\angle ABD = 180° - 35° - 133°$
$= 12°$.

In the triangle ABD, now we know $AD = 30ft$ and it opposite angle $\angle ABD = 12°$. Therefore we can use the law of sine in this triangle to complete y.

$$\frac{\sin 133°}{y} = \frac{\sin 12°}{30}$$

Thus $$y = \frac{30 \sin 12°}{\sin 133°} \approx 105.5285$$

In the triangle ABC, we have $x = y \sin 35° = 105.5285 \times \sin 35°$
≈ 60.5287

The final answer is $x + 6 = 66.5287 \approx 67\ ft$

9) $z = 2+3i$, $w = -1+4i$

A) Review: if $z = a+bi$ and $w = c+di$ then
$$z+w = (a+c)+(b+d)i$$

In other words, the addition is term-by-term, i.e. we perform addition for the real part and imaginary part independently.

In this case $z+w = (2+3i)+(-1+4i) = (2-1)+(3+4)i$
$$= 1+7i$$

B) Review: if $z = a+bi$ and $w = c+di$ then
$$z-w = (a-c)+(b-d)i$$

In other words, we perform the subtraction for the real part and imaginary part independently.

In this case
$$\begin{aligned} z-w &= (2+3i)-(-1+4i) \\ &= (2+1)+(3-4)i \\ &= 3+(-1)i \\ &= 3-i \end{aligned}$$

C) Review: if $z = a+bi$ and $w = c+di$ then $zw \neq (ac)+(bd)i$

This is a common mistake. The multiplication is not performed independently (term-by-term). Instead, we distribute sums as usual and then using the law $i^2 = -1$.

In this case,

$$zw = (2+3i)(-1+4i) = -2 - 3i + 8i + 12i^2$$
$$= -2 - 3i + 8i - 12$$
$$= -14 + 5i$$

D) The common mistake is writing $\dfrac{a+bi}{c+di} = \dfrac{a}{c} + \dfrac{b}{d}i$.

We don't have such an identity!

Instead, we multiply the numerator and denominator by the conjuge of the denominator, i.e. $c-di$.

$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac+bci-adi-bdi^2}{c^2+d^2} = \dots$$

In this case,

$$\frac{z}{w} = \frac{2+3i}{-1+4i} = \frac{(2+3i)(-1-4i)}{(-1+4i)(-1-4i)} = \frac{-2-8i-3i-12i^2}{(-1)^2-16i^2}$$
$$= \frac{-2-8i-3i+12}{1+16}$$
$$= \frac{10-11i}{17}$$
$$= \frac{10}{17} + \left(\frac{-11}{17}\right)i$$

10) $b = 4$, $c = 6$, $B = 40°$

- Using the law of ~~cos~~ sine (here it's impossible to use the law of cosine because B is not the included angle of b and c), we have

$$\frac{\sin 40°}{4} = \frac{\sin C}{6}$$

Thus $\sin C = \frac{6}{4} \sin 40° \approx 0.9641$

Then $C = \sin^{-1} 0.9641 \approx 74.6285°$ or $C = 180° - 74.6285°$
$= 105.3715°$

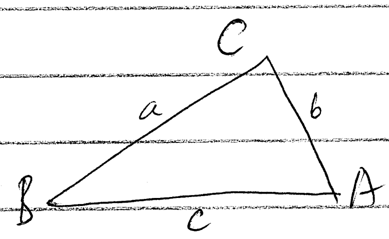
A) For the case $C \approx 74.6°$, we have

- $A = 180° - B - C = 180° - 40° - 74.6° = 65.4°$

- Using the law of sine to find a:

$$\frac{\sin 65.4°}{a} = \frac{\sin 40°}{4}$$

$$\Rightarrow a = \frac{4 \sin 65.4°}{\sin 40°} \approx 5.7$$



- The area is given by

$$\text{Area} = \frac{1}{2} bc \sin A = \frac{1}{2} 4 \cdot 6 \cdot \sin 65.4°$$
$$\approx 10.99$$
$$\approx 11.0$$

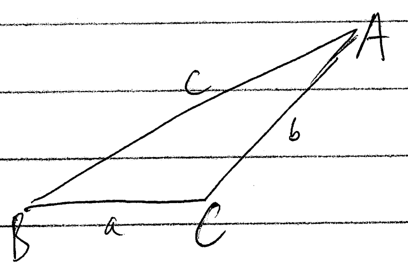
B) For the case $C \approx 105.4°$, we have

- $A = 180° - B - C = 180° - 40° - 105.4° = \cancel{65.4}° 34.6°$

- Using the law of sine to find a:

$$\frac{\sin \cancel{65.4°} 34.6°}{a} = \frac{\sin 40°}{4}$$

$$\Rightarrow a = \frac{4 \sin 34.6°}{\sin 40°} \approx 3.5$$



- The area is given by

$$\text{Area} = \frac{1}{2} bc \sin A = \frac{1}{2} \cdot 4 \cdot 6 \cdot \sin 34.6°$$

$$\approx 6.74$$

$$\approx 6.7$$