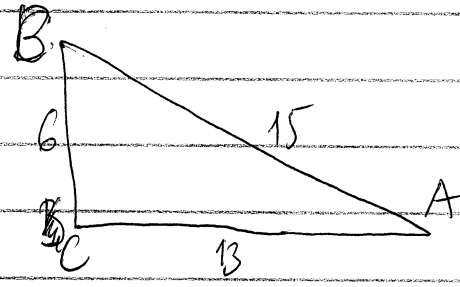
1) This is exactly Question 4 in Quiz 5, which you can find on my website. The answer is: two triangles,

$A = 14.24°$, $C = 155.76°$, $c = 28.37$

$A = 165.76°$, $C = 4.24°$, $c = 5.11$

2) This is exactly Question 4 in Quiz 6, which you can find on my website. The answer is $y = 10$

3)



- Here we're given 3 sides. Thus we can use the law of cosine directly.
- We have

- $$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{15^2+13^2-6^2}{2.15.13} \approx 0.9129$$

Thus $A = \cos^{-1}(0.9129) \approx 23.4°$

- $$\cos B = \frac{c^2+a^2-b^2}{2ca} = \frac{15^2+6^2-13^2}{2.15.6} \approx 0.5111$$

Thus $B = \cos^{-1}(0.5111) \approx 59.4°$

- $$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{6^2+13^2-15^2}{2.6.13} \approx -0.1282$$

Thus $C = \cos^{-1}(-0.1282) \approx 97°.2$

So the best answer among those given is $A = 23.4°$, $B = 59.4°$, $C = 97°.2$

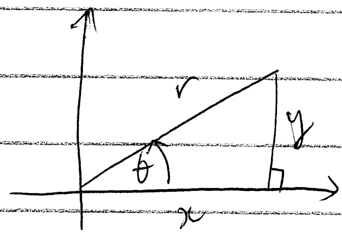
4) $f(x) = 5x^4 - x^2 + 3$

- A potential rational zero of $f(x)$ ~~is~~ of the form p/q where
  - p divides 3, the constant term,
  - q divides 5, the leading coefficient.

- Therefore $p \in \{\pm 1, \pm 3\}$ and $q \in \{1, 5\}$. Note that we can <u>always</u> assume that q is positive because the minus sign of q can be absorbed onto p.

- Then $\frac{p}{q} \in \left\{\pm 1, \pm 3, \pm \frac{1}{5}, \pm \frac{3}{5}\right\}$

5) 、Remember that, to convert (x,y)-coordinates into (r,θ)-coordinate, we use the following formula $\begin{cases} r^2 = x^2+y^2 \\ \sin\theta = \dfrac{y}{r}, \ \cos\theta = \dfrac{x}{r} \end{cases}$



、We are given $(x,y) = (-3,3)$. Thus

$$r = \sqrt{x^2+y^2} = \sqrt{(-3)^2+3^2} = \sqrt{18} = 3\sqrt{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\sin\theta = \frac{y}{r} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

、There are two angles having sine $\frac{1}{\sqrt{2}}$, namely $\frac{\pi}{4}$ and $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

But only one, which is $\frac{3\pi}{4}$, has cosine $-\frac{1}{\sqrt{2}}$.

、Therefore $\theta = \frac{3\pi}{4}$. A The polar coordinate is $\left(3\sqrt{2}, \frac{3\pi}{4}\right)$.

6) $f(x) = x^4 - 12x^2 - 64$

A) The possible rational zeros of $f(x)$ are of the form $\frac{p}{q}$ where

- p is a divisor of -64. $\rightarrow p \in \{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64\}$

- q is a divisor (positive) of 1. $\rightarrow q = 1$

Therefore, all possible rational zeros of $f(x)$ are $\{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64\}$

B) We'll find all real zeros of $f(x)$ by first finding the rational zeros, and then factor $f(x)$.

- We test $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$ for zeros of $f(x)$ by means of long division

$$\begin{array}{r|rrrrr} 4 & 1 & 0 & -12 & 0 & -64 \\ & & 4 & 16 & 16 & 64 \\ \hline & 1 & 4 & 4 & 16 & 0 \end{array}$$

$$\cancel{\begin{array}{rrrrrr} -4 & 1 & 0 & -12 & 0 & -64 \\ & & -4 & 16 & & \end{array}}$$

$$\begin{array}{r|rrrr} -4 & 1 & 4 & 4 & 16 \\ & & -4 & 0 & -16 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

- We see that -4 and 4 are zeros of $f(x)$

- Factor $f(x)$. We have $f(x) = (x-4)(x+4)(x^2+4)$ by looking at the long division above.

- The polynomial x^2+4 has no real zero because $x^2+4>0$.

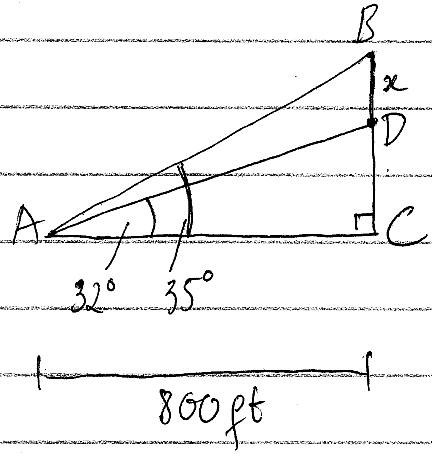
- Therefore, all real zeros of $f(x)$ are -4 and 4.

C) As we see above, the factorization of $f(x)$ over the real system is

$$f(x) = (x-4)(x+4)(x^2+4)$$

The polynomial x^2+4 couldn't be factored over the real system because it has no real zero.

7) This is problem 71, section 8.1, page 515 in the textbook.



- In the right triangle ABC, we have

$$BC = AC\tan 35^\circ = 800\tan 35^\circ$$

- In the right triangle ADC, we have

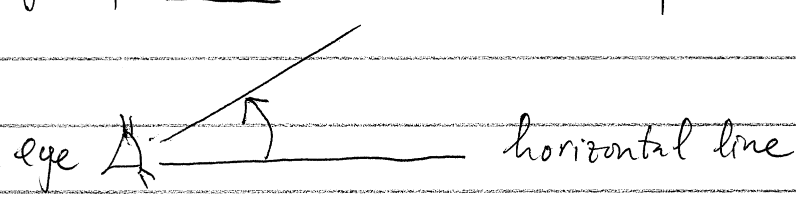
$$CD = AC\tan 32^\circ = 800\tan 32^\circ$$

- Therefore,

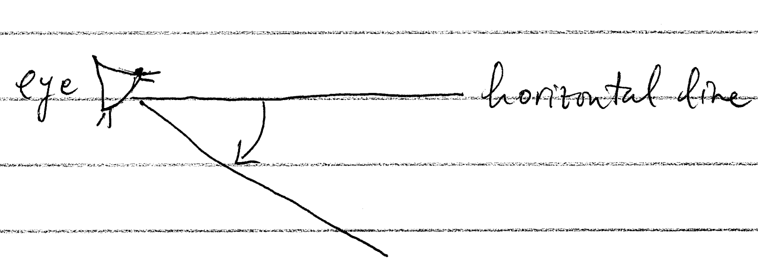
$$x = BC - CD = 800\tan 35^\circ - 800\tan 32^\circ = 800(\tan 35^\circ - \tan 32^\circ) \approx 60.27 \text{ ft}$$

- We round the answer to the nearest foot: $x \approx 60$ ft.

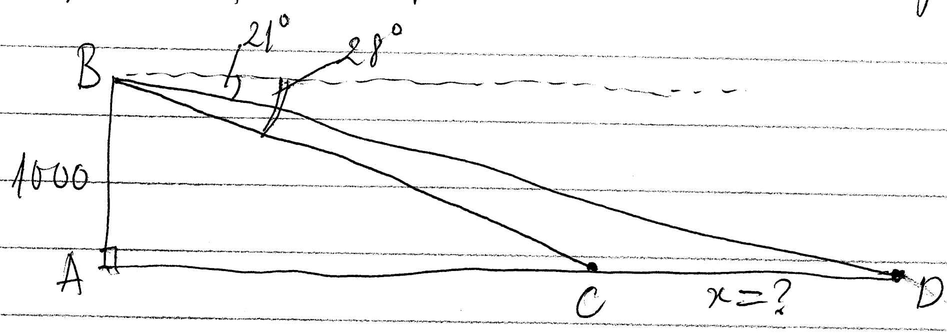
Note
- The angle of elevation measures how "up" our sight is.



- The angle of depression measures how "down" our sight is.



8) The first step is to draw a correct picture.



Now look at the right triangle ABD. We see that

$$\angle ABD = 90^\circ - 21^\circ = 69^\circ$$

Therefore $AD = 1000 \tan 69^\circ$

Now look at the right triangle ABC. We see that

$$\angle ABC = 90^\circ - 28^\circ = 62^\circ$$

Therefore $AC = 1000 \tan 62^\circ$

Thus the distance between two cars is

$$x = AD - AC = 1000 \tan 69^\circ - 1000 \tan 62^\circ$$

$$= 1000 (\tan 69^\circ - \tan 62^\circ)$$

$$\approx 724.36 \text{ ft}$$

We round the answer to the nearest foot $x \approx 724$ ft

9) $z = 3+2i$ and $w = -4+i$

A) Review: if $z = a+bi$ and $w = c+di$ then

$$z+w = (a+c)+(b+d)i$$

In other words, the addition is term-by-term, i.e. we perform addition for the real part and the imaginary part independently.

In this case $z+w = (3+2i)+(-4+i) = (3-4)+(2+1)i$

$$= -1+3i$$

B) Review: if $z = a+bi$ and $w = c+di$ then

$$z-w = (a-c)+(b-d)i$$

In other words, the ~~addition~~ subtraction is term-by-term, i.e. we can subtract the real parts and imaginary parts independently

In this case $z-w = (3+2i)-(-4+i)$

$$= (3+4)+(2-1)i$$

$$= 7+i$$

C) Review: it's a common mistake to assume that

$$zw = ac+bdi$$

This is not true. The multiplication is not performed independently (term-by-term). Instead, we distribute sums as usual and then using the rule $i^2 = -1$.

In this case,

$$zw = (3+2i)(-4+i) = -12-8i+3i+2i^2$$
$$= -12-8i+3i-2$$
$$= -14-5i$$

D) The common mistake is to write $\dfrac{a+bi}{c+di} = \dfrac{a}{c} + \dfrac{b}{d}i$

This is not true. Instead, we multiply the numerator and denominator by the <u>conjugate</u> of the denominator, i.e. $c-di$.

$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac+bci-adi-bdi^2}{c^2+d^2} = \cdots$$

In this case,

$$\frac{z}{w} = \frac{3+2i}{-4+i} = \frac{(3+2i)(-4-i)}{(-4+i)(-4-i)} = \frac{-12-8i-3i-2i^2}{(-4)^2-i^2}$$
$$= \frac{-12-8i-3i+2}{16+1}$$
$$= \frac{-10-11i}{17}$$
$$= -\frac{10}{17} + \left(-\frac{11}{17}\right)i$$

10) $a=6,\ b=8,\ A=35^\circ$

Here it's impossible to use the law of cosine because A is not the angle between a and b. We have to use the law of sine

$$\frac{\sin 35^\circ}{6} = \frac{\sin B}{8}$$

Thus
$$\sin B = \frac{8 \sin 35^\circ}{6} \approx 0.36$$

Then $B = \sin^{-1}(0.36) \approx 49.9^\circ$ or $B = 180^\circ - 49.9^\circ = 130.1^\circ$.

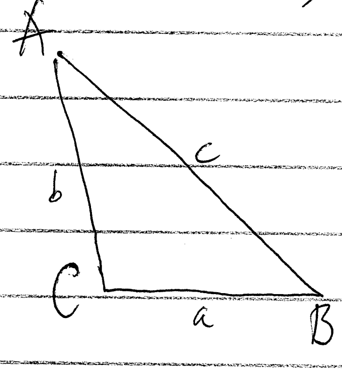
A) For the case $B = 49.9^\circ$, we have

- $C = 180^\circ - A - B = 180^\circ - 35^\circ - 49.9^\circ = 95.1^\circ$

- Using the law of sine to find c:

$$\frac{\sin 95.1^\circ}{c} = \frac{\sin 35^\circ}{6}$$

$$\Rightarrow \quad c = \frac{6 \sin 95.1^\circ}{\sin 35^\circ} \approx 10.4$$



- The area is given by

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} \cdot 6 \cdot 8 \cdot \sin 95.1^\circ$$

$$\approx 23.9$$

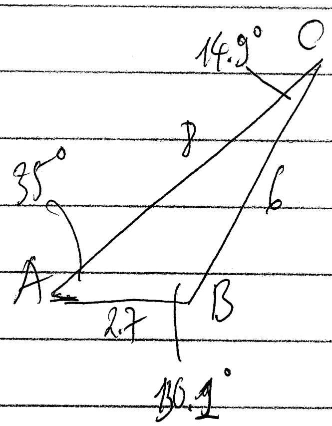
B) For the case $B = 130.1^\circ$, we have

- $C\cancel{A} = 180^\circ - B - A = 180^\circ - 130.1^\circ - 35^\circ = 14.9^\circ$

- Using the law of sine to find c:

$$\frac{\sin 14.9^\circ}{c} = \frac{\sin 35^\circ}{6}$$

$$\Rightarrow c = \frac{6 \sin 14.9^\circ}{\sin 35^\circ} \approx 2.7$$



- The area is given by

$$\text{Area} = \frac{1}{2} ab \sin C = \frac{1}{2} \cdot 6 \cdot 8 \cdot \sin 14.9^\circ$$

$$\approx 6.2$$