KEY

M1151 – Fall 2012 – Exam 4 – November 30, 2012

Name ____

Circle your section (Important!) 31 32 33 34 35

For multiple choice questions (1-5) only your circled answer choice is corrected. For other questions (6-10), be sure to show work and circle your final answer. You may use a non-graphing calculator. If the question asks for an exact answer, that means do not use your calculator. Multiple choice questions are worth 12 points each, and the others are worth 22 points each. Total points possible are 170.

Problem	Possible points	Earned Points
1	12	
2	12	
3	12	
4	12	
5	12	
6	22	
7	22	
8	22	
9	22	
10	22	
	Total	

For the complex numbers $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$,

$$z_1 z_2 = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

For the complex number $z = r(\cos\theta + i\sin\theta)$, $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

Ellipse with major axis parallel to x-axis; $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, a > b > 0, b^2 = a^2 - c^2$

Ellipse with major axis parallel to y-axis; $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, a > b > 0, b^2 = a^2 - c^2$

Hyperbola with major axis parallel to x-axis; $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, b^2 = c^2 - a^2$

Hyperbola with major axis parallel to y-axis; $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1, b^2 = c^2 - a^2$

1) Form a polynomial f(x) with real coefficients having degree 3 and zeros of -4 and 3-2i.

A) $x^3 - 2x^2 - 11x + 52$ B) $x^3 - 2x^2 + 5x - 52$

C) $x^3 - x^2 - 11x + 52$ D) $x^3 - x^2 + 11x + 52$

$(x+4)(x-(3-2i))(x-(3+2i))$

$=(x+4)(x^2-6x+13)$

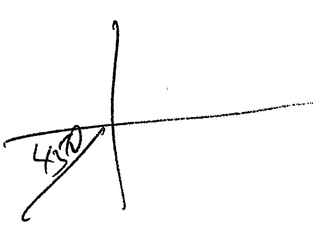
$$= \begin{array}{r} x^3-6x^2+13x \\ +4x^2-24x+52 \\ \hline x^3-2x^2-11x+52 \end{array}$$

2) Write the complex number $[2(\cos 75° + i \sin 75°)]^3$ in standard form.

A) $4\sqrt{2} - 4\sqrt{2}i$ B) $-4\sqrt{2} - 4\sqrt{2}i$ C) $-4 - 4\sqrt{2}i$ D) $4\sqrt{2} + 4\sqrt{2}$

$2^3(\cos 225° + i \sin 225°)$

$= 8\left(-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right) = -4\sqrt{2} - 4\sqrt{2}i$



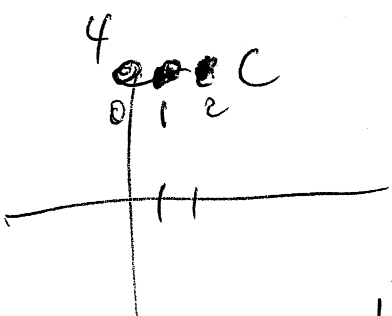
3) A polynomial f(x) is of degree 4, and two of its zeroes are i and 3+i. Find the other complex zeros of f(x).

A) -i, -3+ i B) 3 - i C) -i, 3-i D) -3 + i, 3 – i

4) Find an equation for the hyperbola with center at (2,4), focus at (0,4), and vertex at (1,4).

A) $(x-4)^2 - \frac{(y-2)^2}{3} = 1$ B) $\frac{(x-2)^2}{3} - (y-4)^2 = 1$

C) $\frac{(x-4)^2}{3} - (y-2)^2 = 1$ D) $(x-2)^2 - \frac{(y-4)^2}{3} = 1$



$$\frac{(x-2)^2}{1} - \frac{(y-4)^2}{3} = 1$$

$b^2 = c^2 - a^2$

$b^2 = 2^2 - 1^2 = 3$

$a = 1$

$c = 2$

$b = \sqrt{3}$

5) What conic section does the equation $2x^2+3y^2-8x+6y+5=0$ describe?

A) Parabola

B) Ellipse

C)Hyperbola

D) None of the above

6) Given the polynomial $x^4 - 3x^3 + 5x^2 - x - 10$

A) Find all of the **complex** zeroes of the polynomial. .

$-1, 2, 1 \pm 2i$

$f(-1)=0 \quad f(2)=0$

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & 5 & -1 & -10 \\ & & -1 & 4 & -9 & 10 \\ \hline 2 & 1 & -4 & 9 & -10 & 0 \\ & & 2 & -4 & 10 & \\ \hline & 1 & -2 & 5 & 0 & \end{array}$$

$x^2-2x+5=0 \rightarrow x = \frac{2 \pm \sqrt{4-20}}{2}$

$= 1 \pm \frac{4i}{2} = 1 \pm 2i$

B) Factor the polynomial over the **complex** number system.

$(x+1)(x-2)(x-(1+2i))(x-(1-2i))$

7) For the complex numbers z=2+5i and w= 2-i;

a) Compute z+w, giving the answer in standard form.

$\cancel{4+7i}$ $4+4i$

b) Compute zw, giving the answer in standard form.

$(2+5i)(2-i) = 4-2i+10i-5i^2$

$= 9+8i$

c) Compute z+w, giving the answer in polar form.

$r = \sqrt{16+16} = 4\sqrt{2}$ $\theta = \frac{\pi}{4}$

$(z+w) = 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

d) Compute $(z+w)^{10}$, giving the answer in standard form.

$(4\sqrt{2})^{10}\left(\cos\frac{10\pi}{4} + i\sin\frac{10\pi}{4}\right)$

$= (4\sqrt{2})^{10}(0+i) = (4\sqrt{2})^{10} i$

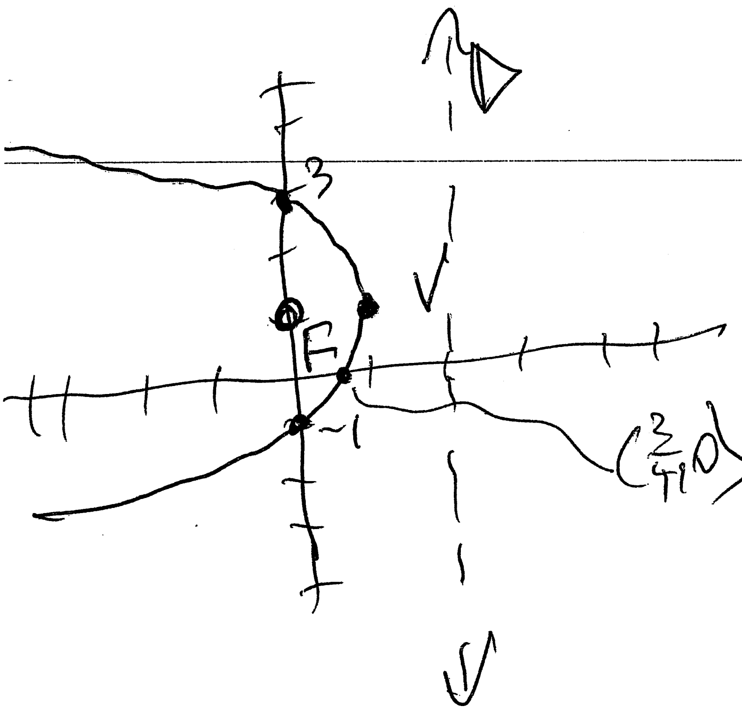
$= 33554432i$

$\frac{10\pi}{4} = \frac{5\pi}{2} = \frac{4\pi}{2} + \frac{\pi}{2}$

either one OK

$a=1$

8) Graph the parabola $(y-1)^2 = -4(x-1)$. Compute the vertex, focus, directrix, x-intercept(s) (if any), and y-intercept(s) (if any). Then graph the parabola, being sure to label all of the points (and the line) listed above.



$V=(1,1)$

$F=(0,1)$

Directrix $x=2$

y-ints:

$(y-1)^2 = 4$

$y-1 = \pm 2$

$y = 1 \pm 2$

$= -1, 3$

y-ints

y-int. $(0,-1), (0,3)$

x-int $(\frac{3}{4}, 0)$

x-ints

$1 = -4(x-1)$

$1 = -4x+4$

$4x = 3$

$x = \frac{3}{4}$

9) $3x^2+4y^2-36x+32y+160=0$ is the equation of an ellipse. Find the center, foci, and vertices exactly. You do **not** need to graph the ellipse, but you may if it helps you to answer the questions.

$$3(x^2-12x)+4(y^2+8)=-160$$

$$3((x-6)^2-36)+4((y+4)^2-16)=-160$$

$$3(x-6)^2-108+4(y+4)^2-64=-160$$

$$3(x-6)^2+4(y+4)^2=12$$

$$\frac{3(x-6)^2}{12}+\frac{4(y+4)^2}{12}=1$$

$$\frac{(x-6)^2}{4}+\frac{(y+4)^2}{3}=1$$

$$\boxed{\begin{array}{l} C=(6,-4) \\ F=(7,-4),(5,-4) \\ V=(8,-4),(4,-4) \end{array}}$$

$$\begin{array}{r} 108 \\ 64 \\ \hline 172 \end{array}$$

$b^2=a^2-c^2$

$c^2=a^2-b^2$

$=4-3=1$

$c=1$

$a=2$

$b=\sqrt{3}$

10) Solve the following systems of equations, remembering that solving a system of equation means there may be one solution, there may be no solutions, or there may be infinitely many solutions.

a)

$$\begin{cases} x + 4y = 5 \\ 5x + 20y = 25 \end{cases}$$

$$\begin{bmatrix} 1 & 4 & 5 \\ 5 & 20 & 25 \end{bmatrix} \to \begin{bmatrix} 1 & 4 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$x+4y=5$

$\to x = 5-4y$

OR $4y = 5 - x$

$y = \frac{5}{4} - \frac{x}{4}$

$\boxed{(5-4y, y)\ y \text{ any real } \#}$

$\boxed{\left(x, \frac{5}{4} - \frac{x}{4}\right)\ x \text{ any real } \#}$

b)

$$\begin{cases} x + y + z = 7 \\ x - y + 2z = 7 \\ 5x + y + z = 11 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 1 & -1 & 2 & 7 \\ 5 & 1 & 1 & 11 \end{bmatrix}$$

$R_2 \leftarrow -R_1 + R_2$

$R_3 \leftarrow -5R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 0 & -2 & 1 & 0 \\ 0 & -4 & -4 & -24 \end{bmatrix}$$

$R_3 \leftarrow -2R_2 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -6 & -24 \end{bmatrix}$$

$R_2 \leftarrow -\frac{1}{2}R_2$

$R_3 \leftarrow -\frac{1}{6}R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$z = 4$

$y = 0 + \frac{1}{2}z$

$= 2$

$x = 7 - y - z$

$= 7 - 2 - 4$

$= 1$

$\boxed{(1, 2, 4)}$

c)

$$\begin{cases} x+y+z &= -1 \\ x-y+2z &= -6 \\ 2x+2y+2z &= -6 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & 2 & -6 \\ 2 & 2 & 2 & -6 \end{bmatrix}$$

$R_2 \leftarrow -R_1 + R_2$

$R_3 \leftarrow -2R_1 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & -2 & 1 & -5 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

contradiction

no solution

or

inconsistent