KEY

M1151 – Fall 2012 – Exam 4 – November 30, 2012

Name ______________________________

Circle your section (Important!) 41 42 43 44 45 46

For multiple choice questions (1-5) only your circled answer choice is corrected.
For other questions (6-10), be sure to show work and circle your final answer.

You may use a non-graphing calculator. If the question asks for an exact answer, that means do not use your calculator. Multiple choice questions are worth 12 points each, and the others are worth 22 points each. Total points possible are 170.

Problem	Possible points	Earned Points
1	12	
2	12	
3	12	
4	12	
5	12	
6	22	
7	22	
8	22	
9	22	
10	22	
	Total	

For the complex numbers $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$,

$$z_1 z_2 = r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$$

For the complex number $z = r(\cos\theta + i\sin\theta)$, $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

Ellipse with major axis parallel to x-axis; $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, a > b > 0, b^2 = a^2 - c^2$

Ellipse with major axis parallel to y-axis; $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, a > b > 0, b^2 = a^2 - c^2$

Hyperbola with major axis parallel to x-axis; $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, b^2 = c^2 - a^2$

Hyperbola with major axis parallel to y-axis; $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1, b^2 = c^2 - a^2$

1) Form a polynomial f(x) with real coefficients having degree 3 and zeros of -2 and 3+i.

A) $x^3 - 6x^2 - 10x + 20$ B) $x^3 - 4x^2 - 10x + 20$

C) $x^3 - 8x^2 + 2x + 20$ D) $x^3 - 4x^2 - 2x + 20$

$(x+2)(x-(3+i))(x-(3-i))$

$(x+2)(x^2-6x+10)$

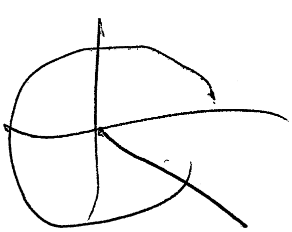
$$\begin{array}{r} x^3-6x^2+10x \qquad \\ +2x^2-12x+20 \\ \hline x^3-4x^2-2x+20 \end{array}$$

2) Write the complex number $[2(\cos 105° + i \sin 105°)]^3$ in standard form.

A) $4 - 4\sqrt{2}i$ B) $4\sqrt{2} - 4\sqrt{2}i$ C) $-4\sqrt{2} + 4\sqrt{2}i$ D) $-4\sqrt{2} - 4\sqrt{2}i$

$2^3(\cos 315° + i \sin 315°)$

$= 8\left(\cancel{\cos}\ \frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$



$= 4\sqrt{2} - 4\sqrt{2}i$

3) A polynomial f(x) is of degree 4, and two of its zeroes are 4-5i and 8i. Find the other complex zeros of f(x).

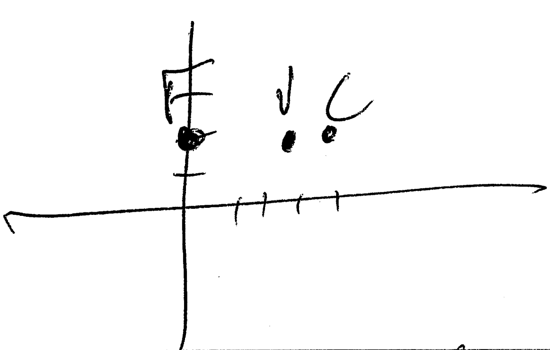
A) -4 - 5i, -8i   B) -4 + 5i, -8i   C) 4 + 5i, 8 - i   D) 4 + 5i, -8i

4) Find an equation for the hyperbola with center at (4,2), focus at (0,2), and vertex at (3,2).

A) $(x-2)^2 - \frac{(y-4)^2}{15} = 1$ B) $\frac{(x-2)^2}{15} - (y-4)^2 = 1$

C) $(x-4)^2 - \frac{(y-2)^2}{15} = 1$ D) $\frac{(x-4)^2}{15} - (y-2)^2 = 1$

$$\frac{(x-4)^2}{1} - \frac{(y-2)^2}{15} = 1$$



$c = 4$

$a = 1$

$b^2 = c^2 - a^2$

$= 16 - 1 = 15$

5) What conic section does the equation $3x^2+4y^2-36x+32y+160=0$ describe?

A) Parabola

B) Ellipse

C)Hyperbola

D) None of the above

$3(x^2-12x)+4(y^2+8y)=-160$

$3((x-6)^2-36)+4((y+4)^2-16)=-160$

$3(x-6)^2-108+4(y+4)^2-64=-160$

$3(x-6)^2+4(y+4)^2=12$

6) Given the polynomial x^4-12x^2-64 ,

A) Find all of the **complex** zeroes of the polynomial. .

$(x^2-16)(x^2+4) \rightarrow x^2=16$ OR $x^2=-4$

$\boxed{x=\pm4, \pm2i}$

easiest way of the possible rational zeros

B) Factor the polynomial over the **complex** number system.

$\boxed{(x-4)(x+4)(x-2i)(x+2i)}$

7) For the complex numbers z=5+2i and w= -2+i;

a) Compute z+w, giving the answer in standard form.

4

$3+3i$

b) Compute zw, giving the answer in standard form.

4

$(5+2i)(-2+i) = -10+5i-4i+2i^2$

$= -12+i$

7 c) Compute z+w, giving the answer in polar form.

$3+3i$

$r=\sqrt{9+9}=3\sqrt{2}$

$\tan^{-1}\frac{3}{3}=\theta=\frac{\pi}{4}$

$3\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

7 d) Compute $(z+w)^{12}$, giving the answer in standard form.

$(3+3i)^{12} = (3\sqrt{2})^{12}\left(\cos\frac{12\pi}{4}+i\sin\frac{12\pi}{4}\right)$

$= (3\sqrt{2})^{12}(\cos 3\pi + i\sin 3\pi)$

$= (3\sqrt{2})^{12}(-1) = -(3\sqrt{2})^{12}$

$= \boxed{-34012224}$

$a=2$

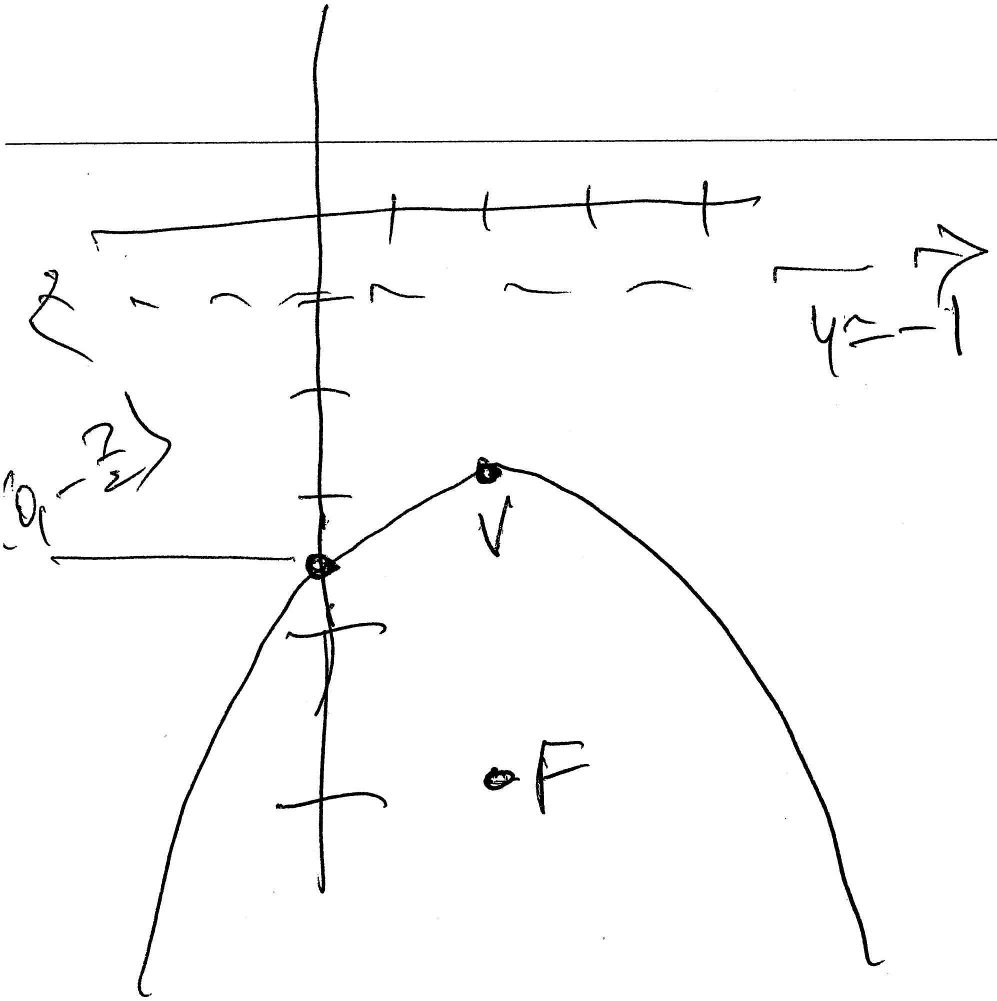
8) Graph the parabola $(x-2)^2 = -8(y+3)$. Compute the vertex, focus, directrix, x-intercept(s) (if any), and y-intercept(s) (if any). Then graph the parabola, being sure to label all of the points (and the line) listed above.

$V=(2,-3)$

$F=(2,-5)$

Directrix

$y=-1$



x-ints

$(x-2)^2 = -24$

so none

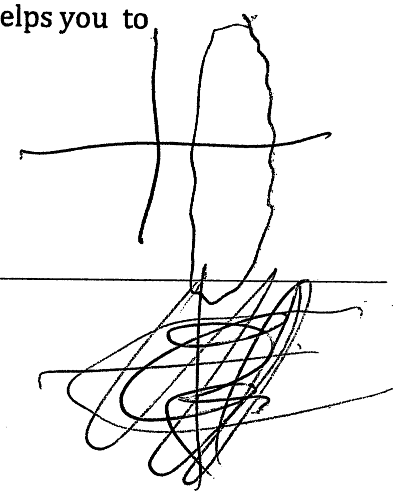
y-ints

$4 = -8y - 24$

$8y = -28$

$y = \frac{-28}{8} = \frac{-7}{2}$

9) $25x^2+y^2-150x+200=0$ is the equation of an ellipse. Find the center, foci, and vertices exactly. You do **not** need to graph the ellipse, but you may if it helps you to answer the questions.



$$25(x^2-6x)+y^2=-200$$

$$25((x-3)^2-9)+y^2=-200$$

$$25(x-3)^2-225+y^2=-200$$

$$25(x-3)^2+y^2=25$$

$$\frac{25(x-3)^2}{25}+\frac{y^2}{25}=1$$

$$\frac{(x-3)^2}{1^2}+\frac{y^2}{5^2}=1$$

$$b^2=a^2-c^2$$

$$c^2=a^2-b^2$$

$$=25-1=24$$

$$c=\sqrt{24}=2\sqrt{6}$$

$$\boxed{\begin{array}{l} C=(3,0) \\ F=(3,2\sqrt{6}), \\ (3,-2\sqrt{6}) \\ V=(3,5), \\ (3,-5) \end{array}}$$

$a=5$

$b=1$

10) Solve the following systems of equations, remembering that solving a system of equation means there may be one solution, there may be no solutions, or there may be infinitely many solutions.

a)

$$\begin{cases} 4x + y = 7 \\ -20x - 5y = -35 \end{cases}$$

$$\begin{bmatrix} 4 & 1 & 7 \\ -20 & -5 & -35 \end{bmatrix}$$

$R_2 \Leftarrow 5R_1 + R_2$

$$\begin{bmatrix} 4 & 1 & 7 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow 4x = 7-y \rightarrow x = \frac{7-y}{4}$$

$$\boxed{\left(\frac{7-y}{4}, y\right) \text{ any } y}$$

or

$$\rightarrow y = 7-4x$$

$$\boxed{(x, 7-4x) \text{ any } x}$$

b)

$$\begin{cases} x - y + z = 8 \\ x + y + z = 6 \\ x + y - z = -12 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 1 & 8 \\ 1 & 1 & 1 & 6 \\ 1 & 1 & -1 & -12 \end{bmatrix}$$

$R_2 \Leftarrow -R_1 + R_2$

$R_3 \Leftarrow -R_1 + R_3$

$$\begin{bmatrix} 1 & -1 & 1 & 8 \\ 0 & +2 & 0 & -2 \\ 0 & 2 & -2 & -20 \end{bmatrix}$$

$R_3 \Leftarrow -R_2 + R_3$

$$\begin{bmatrix} 1 & -1 & 1 & 8 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & -2 & -18 \end{bmatrix}$$

$R_2 \Leftarrow \frac{1}{2} R_2$

$R_3 \Leftarrow -\frac{1}{2} R_3$

$$\begin{bmatrix} 1 & -1 & 1 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 9 \end{bmatrix} \begin{matrix} x = 8 + y - z = -2 \\ \rightarrow y = -1 \\ \rightarrow z = 9 \end{matrix}$$

$$\boxed{(-2, -1, 9)}$$

c)

$$\begin{cases} x - y + 2z = 1 \\ 5x + z \qquad = 0 \\ -x + y - 2z \;=\; -2 \end{cases}$$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 5 & 0 & 1 & 0 \\ -1 & 1 & -2 & -2 \end{bmatrix}$$

$R_2 \leftarrow -5R_1 + R_2$

$R_3 \leftarrow R_1 + R_3$

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 5 & -9 & -5 \\ 0 & 0 & 0 & -1 \end{bmatrix} \leftarrow \text{contradiction}$$

(no solutions

OR

inconsistent