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Math 1151, Th 11/15 notes.

Tomorrow: Quiz 7: 4.6 & 9.3 (4.6 mostly)

Monday next week: Quiz 8: 9.3 & 10.2

} all are multiple choice

Today: Review for 4.6 & 9.3. Send remarks of 10.2 in weekend.

<u>4.6</u> Study the complex zeros of a polynomial, specifically polynomials with <u>real</u> coefficients.

<u>Given a polynomial $f(x)$. Goal: factor $f(x)$</u>

- Consider x^2+x+1. $f(x)$ has no real zeros since $\Delta=-3<0$.
- Complex zeros $x=-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$
- $x^2+x+1=\left[x-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\right]\left[x-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]$ - factorization

<u>Important fact</u>: Given a polynomial with real coefs. If $a+bi$ is a zero, then so is $a-bi$.

Why?

$f(x)=4x^4+2x^2+1$

$r=a+bi$ is a zero. $f(r)=4r^4+2r^2+1=0$

$f(\bar{r})=4\bar{r}^4+2\bar{r}^2+1=\overline{4r^4+2r^2+1}=\bar{0}=0$

<u>Reminder</u>:
- Conjugate of a product is the product of conjugates.
- " sum " sum ".

Ex: Given a poly. with real coeffs. $f(x)$ of degree 4. Two of its zeros are $1-2i$ and $3i$. Find the other zeros.

- How many zeros are there?
  - $1-2i$ is a zero $\Rightarrow$ $1+2i$ is a zero
  - $3i$ " $\Rightarrow$ $-3i$ "
  - Four zeros: $1 \pm 2i$, $\pm 3i$

Ex: Given $f(x)$ with real coeffs, of degree 6. Two of its zeros are $1-i$ and $2i$. Find the other zeros.

- Can find only four: $1 \pm i$, $\pm 2i$
- Not enough information.

Form a polynomial from its zeros

Facts:

- A poly. $f(x)$ of degree n has exactly n c[omplex zeros]
- If $f(x)$ is of deg. n and has n zeros $x_1, \dots, x_n$ then

$$f(x) = a(x-x_1)\dots(x-x_n)$$

Ex: Form a poly. of degree 3 having zeros 1, 2, 3

$$f(x) = a(x-1)(x-2)(x-3)$$

Ex: Form a poly. of degree 2 having a zero $1+i$

- $1-i$ is also a zero.
- $(1+i)+(1-i) = 2$, $(1+i)(1-i) = 1^2 - i^2 = [2]$
- $x^2 - 2x + 2$

Ex: Form a poly. of degree 3 having zeros 1, $2+i$

- $2-i$ is also a zero

- $f(x) = (x-1)\underbrace{(x-(2+i))(x-(2-i))}_{\text{reduce by the same technique above}}$

- $f(x) = (x-1)(x^2-4x+5)$

Ex: One of the zero of $2x^3+3x^2+2x-2$ ~~is~~ $1+i$. Find other zeros

- $1-i$ is also a zero. Need one more!

- $f(x)$ has a factor $(x-(1-i))(x-(1+i)) = x^2-2x+2$

- $f(x) = (x^2-2x+2)$ ← something

- Look at the leading coefficient and the constant term:
  $f(x) = (x^2-2x+2)(2x-1)$

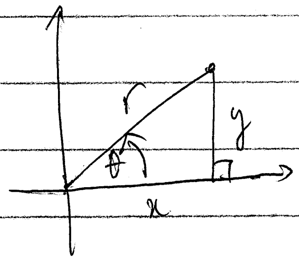
- The third zero is $x = \frac{1}{2}$.

9.3 Arithmetic operations on complex numbers

addition }
subtraction } easy to use rectangular form

$$\left.\begin{aligned}(a+bi)+(c+di) &= (a+c)+(b+d)i\\ (a+bi)-(c+di) &= (a-c)+(b-d)i\end{aligned}\right\}\text{ term by term}$$

$$\left.\begin{array}{l}\text{multiplication}\\ \text{power}\\ \text{division}\end{array}\right\}\text{ easy to use polar form}$$



Remind : $r = \sqrt{x^2+y^2}$

$$\begin{cases}\cos\theta = \dfrac{x}{r}\\ \sin\theta = \dfrac{y}{r}\end{cases}$$

Ex

Write $2+2i$ in polar form

- $x = 2,\ y = 2$
- $r = \sqrt{x^2+y^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$
- $\begin{cases}\cos\theta = x/r = 2/2\sqrt{2} = 1/\sqrt{2} = \sqrt{2}/2\\ \sin\theta = y/r = 2/2\sqrt{2} = \sqrt{2}/2\end{cases} \Rightarrow \theta = \dfrac{\pi}{4}$
- $2+2i = r(\cos\theta + i\sin\theta) = 2\sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$