Sun, 11/18 – Remarks for Quiz 8

- Section 9.3 & 10.2

no finding-root problems

no word-problems

$\boxed{9.3}$

- For addition & substraction: convenient to use rectangular form;

$$(a+bi)+(c+di) = (a+c)+(b+d)i,$$

$$(a+bi)-(c+di) = (a-c)+(b-d)i.$$

- For multiplication & division: convenient to use polar form

$$z_1 = r_1(\cos\theta_1 + i\sin\theta_1),$$

$$z_2 = r_2(\cos\theta_2 + i\sin\theta_2),$$

$$\Rightarrow \quad z_1 z_2 = r_1 r_2\big(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\big), \quad \text{and}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\big(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\big)$$

- De Moivre's theorem: obtained by multiplying many times.

$$z^n = \underbrace{z \cdot z \cdot \cdots z}_{n\text{ times}} = \underbrace{r \cdot r \cdots r}_{n\text{ times}}\Big(\cos(\underbrace{\theta + \cdots + \theta}_{n\text{ times}}) + i\sin(\underbrace{\theta + \cdots + \theta}_{n\text{ times}})\Big)$$

$$= r^n\big(\cos(n\theta) + i\sin(n\theta)\big)$$

Ex Compute $z = [2(\cos 75^\circ + i\sin 75^\circ)]^3$

Using De Moivre's theorem:

$$z = 2^3\left(\cos(3\times 75^\circ) + i\sin(3\times 75^\circ)\right)$$

$$= 8\left(\cos 225^\circ + i\sin 225^\circ\right)$$

Because $225^\circ = 180^\circ + 45^\circ$, we have

$$\cos 225^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

$$\sin 225^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$$

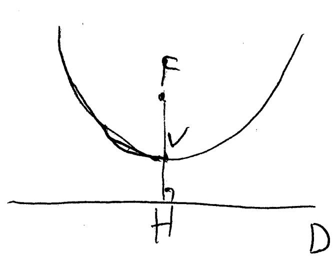
$$\Rightarrow z = 8\left(-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right) = -8\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = -4\sqrt{2} - i\,4\sqrt{2}$$

Ex Problem 42, page 588.

Answer: $-\frac{27}{2} - \frac{27\sqrt{3}}{2}i$

10.2 Parabola

- Description: collection of all points that are of the same distance from a specified point, called focus, and a specified line, called directrix.



From F, we draw a line perpendicular to D. The intersect is labeled H. Then the midpoint of FH is called the vertex of the parabola.

Two kinds of problems:

1. Given the vertex and the focus. Find the equation of the parabola.

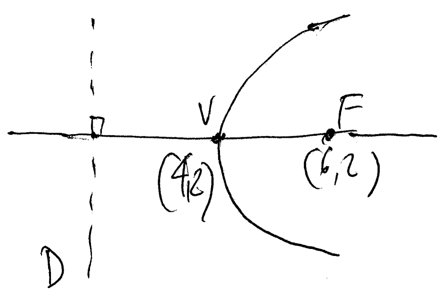
or ________ focus ________ directrix ________________.

or ________ vertex ________ directrix ________________.

Ex Vertex at $(4,2)$, focus at $(6,2)$

Step 1 draw these points. The directrix can be found by taking the reflection of F with respect to V. Then draw a rough shape of the parabola to see if it's open up, down, left or right.

In our case, the parabola is open right.



Step 2 The coefficient a is the distance from V to F. Thus, $a = 2$.

Step 3 Because the parabola is open right, it has the form "$y^2 = \dots$".

$$(y-2)^2 = 4a(x-4)$$

Coordinates of the vertex

Thus, $\boxed{(y-2)^2 = 8(x-4)}$

⚠ Remark: if the parabola is open left, we will write $(y - \dots)^2 = -4a(x - \dots)$

the minus sign

Key to Problems 31 → 36:

31) $(y+2)^2 = 4(x+1)$

32) $(x-3)^2 = -8y$

33) $(x+3)^2 = 4(y-3)$

34) $(y-4)^2 = 12(x+1)$

35) $(y+2)^2 = -8(x+1)$

36) $(x+4)^2 = 12(y-1)$

Key to Problems 39, 42, 44, 46, 47

39) $V(0,0)$; $F(-4,0)$; $D: x=4$

42) $V(-4,-2)$; $F(-4,2)$; $D: y=-6$

44) $V(2,-1)$; $F(1,-1)$; $D: x=3$

46) $V(2,3)$; $F(2,4)$; $D: y=2$

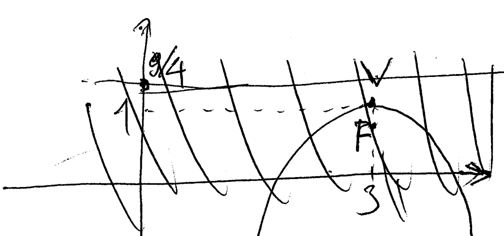
47) $V(0,2)$; $F(-1,2)$; $D: x=1$

(2) Find vertex, focus, directrix from the equation.

Ex: Problem 43

$$(x-3)^2 = -(y+1)$$

- By making both side equal 0, we get the vertex $(3,-1)$.
- The coefficient a is $a = \frac{1}{4}$. This is the distance from the vertex to the focus.
- Because the equation is of the form "$x^2 = ..$", the parabola is open down.



$$F = \left(3, -1-\frac{1}{4}\right) = \left(3, -\frac{5}{4}\right)$$

$$D: y = -1+\frac{1}{4} = -\frac{3}{4}$$