

1) $\vee f(x)$ is of degree 4 \Rightarrow it has exactly 4 complex zeros.

$\vee f(x)$ has real coefficients \Rightarrow if z is a zero, then \bar{z} is also a zero.

$\vee 4-5i$ is a zero $\Rightarrow \overline{4-5i} = 4+5i$ is a zero.

$\vee 8i$ is a zero $\Rightarrow \overline{8i} = -8i$ is a zero.

(D)

2) $\vee f(x)$ is of degree 3 \Rightarrow it has exactly 3 complex zeros.

\vee Since $3-2i$ is a zero, $\overline{3-2i} = 3+2i$ is also a zero.

$$\vee f(x) = (x+4) \left(x - \underbrace{(3-2i)}_a \right) \left(x - \underbrace{(3+2i)}_b \right)$$

Trick: $(x-a)(x-b) = x^2 - (a+b)x + ab$

$$\begin{cases} a+b = (3-2i) + (3+2i) = 6 \\ ab = (3-2i)(3+2i) = 3^2 - (2i)^2 = 9+4 = 13 \end{cases}$$

$$\vee f(x) = (x+4)(x^2 - 6x + 13)$$

$$= x^3 - 2x^2 - 11x + 52$$

(A)

$$3) \setminus f(x) = x^3 + 7x^2 - 16x + 18$$

$\setminus 1+i$ is a zero $\Rightarrow 1-i$ is also a zero.

$\setminus f(x)$ has a factor $(x - \underbrace{(1+i)}_a)(x - \underbrace{(1-i)}_b)$

$$a+b = (1+i) + (1-i) = 2$$

$$ab = (1+i)(1-i) = 1^2 - i^2 = 2$$

$$= x^2 - 2x + 2$$

\setminus Using long div $f(x) = (x^2 - 2x + 2)(\dots)$

\setminus Comparing the leading coeff, then the constant terms, we get

$$f(x) = (x^2 - 2x + 2)(x^2 + 9)$$

\Rightarrow the third zero is -9 .

(A)

$$4) \quad 2+2i$$

$$\vee \quad x=2, \quad y=2$$

$$\vee \quad r = \sqrt{x^2 + y^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\vee \quad \cos \theta = \frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sin \theta = \frac{y}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

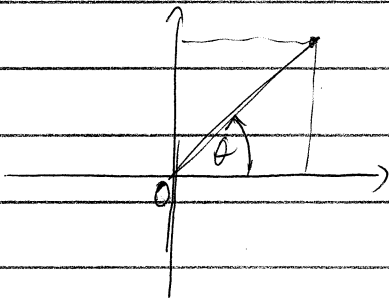
$$\Rightarrow \theta = \frac{\pi}{4} \quad (=45^\circ)$$

$$\vee \quad 2+2i = 2\sqrt{2} (\cos 45^\circ + i \sin 45^\circ)$$

(C)

Remark: we can use $\tan \theta = \frac{y}{x} = \frac{2}{2} = 1$

$$\Rightarrow \theta = 45^\circ + k \cdot 180^\circ$$



Choose $k=1$ because the point is
in the first quadrant.