

1) $\vee f(x)$ is of degree 4 \Rightarrow it has exactly 4 complex zeros.

$\vee f(x)$ has real coefficients \Rightarrow if z is a zero, then \bar{z} is also a zero.

$\vee i$ is a zero $\Rightarrow \bar{i} = -i$ is also a zero.

$\vee 3+i$ is a zero $\Rightarrow \overline{3+i} = 3-i$ is also a zero.

(C)

2) $\vee f(x)$ is of degree 3 \Rightarrow it has exactly 3 complex zeros

\vee Since $3+i$ is a zero, $3-i$ is also a zero.

$$\vee f(x) = (x+2) \left(x - \underbrace{(3+i)}_a \right) \left(x - \underbrace{(3-i)}_b \right)$$

Trick: $(x-a)(x-b) = x^2 - (a+b)x + ab$

$$a+b = (3+i) + (3-i) = 6$$

$$ab = (3+i)(3-i) = 3^2 - i^2 = 10$$

$$\vee f(x) = (x+2)(x^2 - 6x + 10)$$

$$= x^3 - 4x^2 - 2x + 20$$

(D)

$$3) \quad f(x) = x^3 - 2x^2 - 11x + 52$$

$\therefore -4$ is a zero of $f(x)$. $\Rightarrow f(x)$ has a factor $x+4$.

\therefore Using long division, we get

$$\begin{array}{r|rrrr} -4 & 1 & -2 & -11 & 52 \\ & & -4 & 24 & -52 \\ \hline & 1 & -6 & 13 & 0 \end{array}$$

$$\therefore f(x) = (x+4)(x^2 - 6x + 13)$$

\therefore Find the zeros of $x^2 - 6x + 13$.

$$\Delta = 6^2 - 4 \cdot 13 = 36 - 52 = -16$$

$\therefore x^2 - 6x + 13$ has two distinct complex zeros

$$x = \frac{6 \pm i\sqrt{16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

(B)

$$4) \quad 2+2i$$

$$\therefore x=2, \quad y=2$$

$$\therefore r = \sqrt{x^2 + y^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\therefore \cos \theta = \frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\sin \theta = \frac{y}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

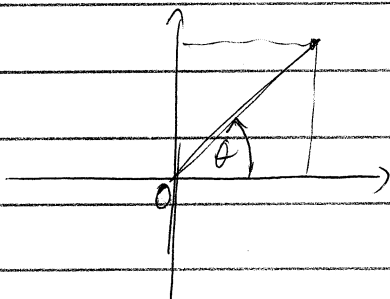
$$\Rightarrow \theta = \frac{\pi}{4} \quad (=45^\circ)$$

$$\therefore 2+2i = 2\sqrt{2} (\cos 45^\circ + i \sin 45^\circ)$$

(C)

Remark: we can use $\tan \theta = \frac{y}{x} = \frac{2}{2} = 1$

$$\Rightarrow \theta = 45^\circ + k \cdot 180^\circ$$



Choose $k=1$ because the point is
in the first quadrant.