

$$1) \quad z = 8 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$w = 3 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

$$\Rightarrow zw = 8 \cdot 3 \left(\cos \left(\frac{\pi}{6} + \frac{\pi}{2} \right) + i \sin \left(\frac{\pi}{6} + \frac{\pi}{2} \right) \right)$$

$$= 24 \left(\cos \frac{4\pi}{6} + i \sin \frac{4\pi}{6} \right)$$

$$= 24 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right)$$

$$2) \quad [2(\cos 75^\circ + i \sin 75^\circ)]^3 = 2^3 (\cos(3 \cdot 75^\circ) + i \sin(3 \cdot 75^\circ)) \quad (\text{De Moivre})$$

$$= 8 (\cos 225^\circ + i \sin 225^\circ)$$

Because $225^\circ = 180^\circ + 45^\circ$, we have

$$\cos 225^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$$

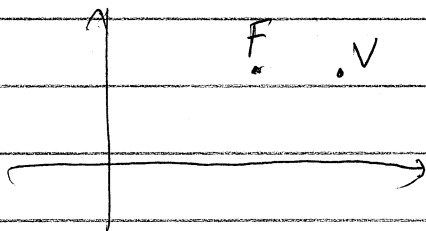
$$\sin 225^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$$

Therefore,

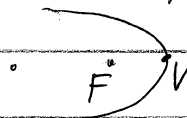
$$\text{the result is } 8 \left(-\frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2} \right) = -4\sqrt{2} - 4\sqrt{2}i$$

3) vertex = $(8, 6)$ and focus = $(3, 6)$

Draw these two points in the plane:



We see that the focus lies on the left of the vertex. Thus, the parabola is open left.



The equation of the parabola looks like " $y^2 = \dots$ "

Since $V = (8, 6)$, it should have the form

$$(y - 6)^2 = -4a(x - 8)$$

the minus sign is because the parabola is open left.

a is the distance from V to F , which is 5.

Therefore the equation is $(y - 6)^2 = -20(x - 8)$.

$$4) (y+2)^2 = 12(x-4)$$

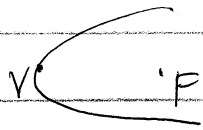
By setting both sides zero, we get $x=4$ and $y=-2$. Thus the vertex

is $(4, -2)$.

The distance between the vertex and the focus is $a = \frac{12}{4} = 3$.

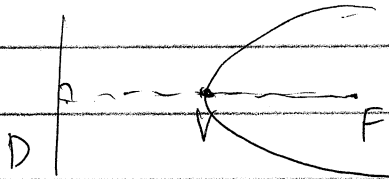
Because the equation is of the form " $y^2 = \dots$ ", the parabola is open left or right.

Because we see the plus sign in front of 12, the parabola is open right.



Therefore, $F = (4 + 3, -2) = (7, -2)$

The directrix is obtained by taking the reflection of F with respect to V.



Therefore the equation of D is $x = 4 - 3 = 1$

In short,

$$\begin{array}{l} V = (4, -2) \\ F = (7, -2) \\ D: x = 1 \end{array}$$