

$$1) \quad z = 6 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

$$w = 12 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$\Rightarrow zw = 6 \cdot 12 \left(\cos \left(\frac{3\pi}{2} + \frac{5\pi}{6} \right) + i \sin \left(\frac{3\pi}{2} + \frac{5\pi}{6} \right) \right)$$

$$= 72 \left(\cos \frac{14\pi}{6} + i \sin \frac{14\pi}{6} \right)$$

We see that $\frac{14\pi}{6} = \frac{12\pi + 2\pi}{6} = 2\pi + \frac{2\pi}{6} = 2\pi + \frac{\pi}{3}$

Therefore, $zw = 72 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$

$$2) \quad [2 (\cos 105^\circ + i \sin 105^\circ)]^3 \stackrel{\text{De Moivre}}{=} 2^3 (\cos(3 \cdot 105^\circ) + i \sin(3 \cdot 105^\circ))$$

$$= 8 (\cos 315^\circ + i \sin 315^\circ) \quad (*)$$

Because $315^\circ = 360^\circ - 45^\circ$, we have

$$\cos 315^\circ = \cos(360^\circ - 45^\circ) = \cos(-45^\circ) = \cos 45^\circ = \frac{\sqrt{2}}{2}$$

↑ cosine is even

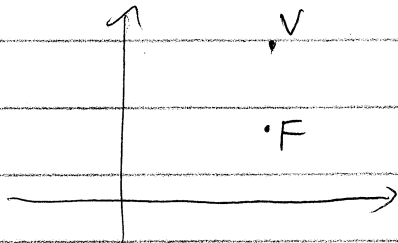
$$\sin 315^\circ = \sin(360^\circ - 45^\circ) = \sin(-45^\circ) = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$$

↑ sine is odd

Therefore $(*) = 8 \left(\frac{\sqrt{2}}{2} - i \frac{\sqrt{2}}{2} \right) = 4\sqrt{2} - i4\sqrt{2}$

3) vertex = $(8, 7)$ and focus = $(8, 3)$

↳ Draw these two points on the plane:



We see that the focus lies below the vertex. Thus, the parabola is open down



↳ The equation of the parabola will have the form " $x^2 =$ ".

↳ Since $V = (8, 7)$, it should have the form $(x - 8)^2 = -4a(y - 7)$

the minus sign is because the parabola is open down.

↳ a is the distance from V to F , which is 4 .

↳ Therefore the equation is $(x - 8)^2 = -16(y - 7)$

$$4) (y+2)^2 = -16(x+1)$$

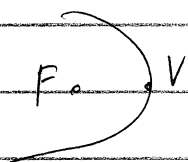
By setting both sides zero, we get $x = -1$ and $y = -2$. Thus

the vertex is $(-1, -2)$.

The distance between the vertex and the focus is $a = \frac{16}{4} = 4$.

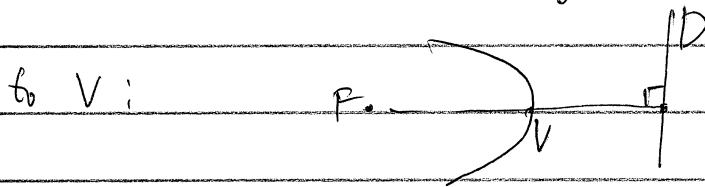
Because the equation is of the form " $y^2 = \dots$ ", the parabola is open left or right.

Because we see the minus sign before 16, the parabola is open left.



Therefore, $F = (-1 - 4, -2) = (-5, -2)$

The directrix is obtained by taking the reflection of F with respect



Therefore the equation of D is $x = -1 + \frac{16}{4} = 3$

In short,

$V = (-1, -2)$
$F = (-5, -2)$
$D: x = 3$