Quiz 9

(1) $\begin{cases} 2x - 9y - z = -23 \\ x + 9y - 5z = 2 \\ 3x + y + z = 24 \end{cases}$

\ Augmented matrix $\left(\begin{array}{ccc|c} 2 & -9 & -1 & -23 \\ 1 & 9 & -5 & 2 \\ 3 & 1 & 1 & 24 \end{array}\right)$

\ Transform it into a row echelon form

$$\xrightarrow[R_3 \to r_3 - 3r_2]{R_1 \to r_1 - 2r_2} \left(\begin{array}{ccc|c} 0 & -27 & 9 & -27 \\ 1 & 9 & -5 & 2 \\ 0 & -26 & 16 & 18 \end{array}\right)$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 9 & -5 & 2 \\ 0 & -27 & 9 & -27 \\ 0 & -26 & 16 & 18 \end{array}\right)$$

$$\xrightarrow{R_2 = -R_2 + R_3} \left(\begin{array}{ccc|c} 1 & 9 & -5 & 2 \\ 0 & 1 & 7 & 45 \\ 0 & -26 & 16 & 18 \end{array}\right)$$

$$\xrightarrow{R_3 \to R_3 + 26R_2} \left(\begin{array}{ccc|c} 1 & 9 & -5 & 2 \\ 0 & 1 & 7 & 45 \\ 0 & 0 & 198 & 1188 \end{array}\right)$$

$$\xrightarrow{R_3 \to \frac{r_3}{198}} \left(\begin{array}{ccc|c} 1 & 9 & -5 & 2 \\ 0 & 1 & 7 & 45 \\ 0 & 0 & 1 & 6 \end{array}\right) \leftarrow \text{echelon form!}$$

- Rewrite the system

$$\begin{cases} x + 9y - 5z = 2 & \Rightarrow x = 2 + 5z - 9y = 2 + 30 - 27 = 5 \\ y + 7z = 45 & \Rightarrow y = 45 - 7z = 45 - 7.6 = 3 \\ z = 6 \end{cases}$$

Therefore $x = 5$, $y = 3$, $z = 6$.

B

2

$$\begin{cases} x^2 - y = -3 & (1) \\ \frac{x^2}{49} + \frac{y^2}{9} = 1 & (2) \end{cases}$$

- $(1) \Rightarrow x^2 = y - 3$

- Substituting x^2 by $y-3$ in (2), we get

$$\frac{y-3}{49} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{49y^2 + 9(y-3)}{49 \times 9} = 1$$

$$\Rightarrow 49y^2 + 9y - 27 = 441$$

$$\Rightarrow 49y^2 + 9y - 468 = 0$$

$$\Delta = 9^2 - 4.49.(-468) = 91809 \Rightarrow \sqrt{\Delta} = 303$$

$$\Rightarrow y = \frac{-9 \pm 303}{2 \times 49}$$

For plus sign, $y = \dfrac{-9+303}{98} = 3$

For minus sign, $y = \dfrac{-9-303}{98} = -17.33\ldots$

The value $y = -17.33\ldots$ doesn't give any x because if so

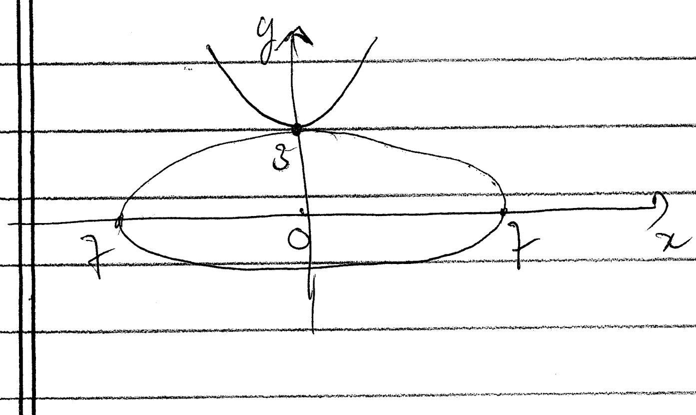
$$x^2 = y - 3 = -20.33\ldots < 0 \quad \text{(impossible)}$$

Thus $y = 3$, which gives $x^2 = 0$ $(\Rightarrow x = 0)$.

$\Rightarrow$ There exists only one solution. B

Second approach: using graph

The first equation is equivalent to $y = x^2 + 3$ (parabola).
The second equation is equivalent to $\dfrac{x^2}{7^2} + \dfrac{y^2}{3^2} = 1$ (ellipse)



The unique intercept is at $(0,3)$. Therefore, the system has only one solution.

(3) $\begin{cases} x^2 + y^2 = 145 \\ x^2 - y^2 = 17 \end{cases}$

Put $u = x^2$, $v = y^2$. We get

$$\begin{cases} u + v = 145 \\ u - v = 17 \end{cases}$$

- Sum them up: $2u = 145 + 17 = 162$
$\Rightarrow u = 81$

- Subtract the second equation from the first: $2v = 128$
$\Rightarrow v = 64$

- $\begin{cases} x^2 = 81 \\ y^2 = 64 \end{cases}$

- This is equivalent to $x = \pm 9$, $y = \pm 8$.

- There are four solutions: $(9, 8)$, $(9, -8)$, $(-9, 8)$, $(-9, -8)$.

④ $\begin{cases} x - y = 6 \\ x + y = 7 \end{cases}$

- Augmented matrix $\left(\begin{array}{cc|c} 1 & -1 & 6 \\ 1 & 1 & 7 \end{array}\right)$

- Transform it into a row echelon form

$$\left(\begin{array}{cc|c} 1 & -1 & 6 \\ 1 & 1 & 7 \end{array}\right) \xrightarrow{R_2 \to r_2 - r_1} \left(\begin{array}{cc|c} 1 & -1 & 6 \\ 0 & 2 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 = \frac{r_2}{2}} \left(\begin{array}{cc|c} 1 & -1 & 6 \\ 0 & 1 & 1/2 \end{array}\right)$$ ← echelon form!

- Rewrite the system

$\begin{cases} x - y = 6 \\ y = 1/2 \end{cases}$ $\Rightarrow x = 6 + y = 6 + 1/2 = 13/2$

$\Rightarrow (x, y) = \left(\frac{13}{2}, \frac{1}{2}\right)$