Quiz 9

① $\begin{cases} 4x - y - 7z = 3 \\ -8x \quad\quad - 6z = -88 \\ \quad\quad 7y + z = 11 \end{cases}$

› Write the augmented matrix

$$\left(\begin{array}{ccc|c} 4 & -1 & -7 & 3 \\ -8 & 0 & -6 & -88 \\ 0 & 7 & 1 & 11 \end{array}\right)$$

› Transform it into the echelon form

$$\xrightarrow[R_1 = r_1/4]{R_2 = r_2 + 2r_1} \left(\begin{array}{ccc|c} 1 & -1/4 & -7/4 & 3/4 \\ 0 & -2 & -20 & -82 \\ 0 & 7 & 1 & 11 \end{array}\right)$$

$$\xrightarrow{R_2 \to r_2/(-2)} \left(\begin{array}{ccc|c} 1 & -1/4 & -7/4 & 3/4 \\ 0 & 1 & 10 & 41 \\ 0 & 7 & 1 & 11 \end{array}\right)$$

$$\xrightarrow{R_3 \to r_3 - 7r_2} \left(\begin{array}{ccc|c} 1 & -1/4 & -7/4 & 3/4 \\ 0 & 1 & 10 & 41 \\ 0 & 0 & -69 & -276 \end{array}\right)$$

$$\xrightarrow{R_3 \to \frac{r_3}{-69}} \left(\begin{array}{ccc|c} 1 & -1/4 & -7/4 & 3/4 \\ 0 & 1 & 10 & 41 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

› Rewrite the system

$$\begin{cases} x - \dfrac{y}{4} - \dfrac{7z}{4} = \dfrac{3}{4} & (*) \\ y + 10z = 41 & \Rightarrow\ y = 41 - 10\times 4 = 1 \\ z = 4 \end{cases}$$

$(*) \Rightarrow x = \dfrac{y}{4} + \dfrac{7z}{4} + \dfrac{3}{4} = \dfrac{1}{4} + \dfrac{28}{4} + \dfrac{3}{4} = 8$

Therefore $x = 8,\ y = 1,\ z = 4$ D

② $\begin{cases} x^2 - y = -2 & (1) \\ \dfrac{x^2}{49} + \dfrac{y^2}{9} = 1 & (2) \end{cases}$

\ $(1) \Rightarrow x^2 = y - 2$

\ Replace x^2 by $y-2$ in (2), we get

$$\frac{y-2}{49} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{9(y-2) + 49y^2}{441} = 1$$

$$\Rightarrow 49y^2 + 9y - 18 = 441$$

$$\Rightarrow 49y^2 + 9y - 460 = 0$$

$$\Delta = 9^2 - 4\times 49 \times (-460) = 90241$$

$$\Rightarrow \sqrt{\Delta} \approx 300$$

$\Rightarrow \quad y \approx \dfrac{-9 \pm 300}{2 \times 49}$

For plus sign, $\quad y \approx 3$

For minus sign, $\quad y \approx -3$

Then $x^2 = y - 2$ $\begin{cases} \approx 3 - 2 = 1 & \text{for } y \approx 3 \\ \approx -3 - 2 = -5 & \text{for } y \approx -3 \end{cases}$

impossible because $x^2 \geq 0$

$\Rightarrow \quad x^2 \approx 1, \; y \approx 3$

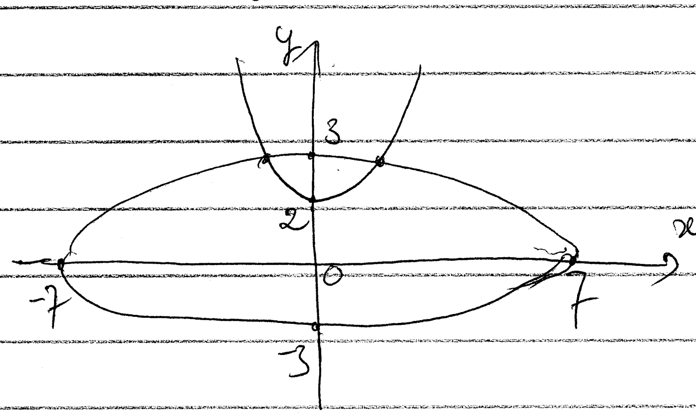
$\Rightarrow \quad x \approx \pm 1, \; y \approx 3$

$\Rightarrow$ the system has <u>two</u> solutions (C)

Second approach : using graph

$x^2 - y = -2 \;\Rightarrow\; y = x^2 + 2$ (parabola)

$\dfrac{x^2}{49} + \dfrac{y^2}{9} = 1 \;\Rightarrow\; \dfrac{x^2}{7^2} + \dfrac{y^2}{3^2} = 1$ (ellipse)



There are two intercepts.

$\Rightarrow$ The system has <u>two</u> solutions

(3) $\begin{cases} 3x^2 - 3y^2 = -15 \\ 5x^2 + 4y^2 = 56 \end{cases}$

- Put $u = x^2$, $v = y^2$

$$\begin{cases} 3u - 3v = -15 \\ 5u + 4v = 56 \end{cases}$$

- Augmented matrix $\left(\begin{array}{cc|c} 3 & -3 & -15 \\ 5 & 4 & 56 \end{array}\right)$

- Transform it into a row echelon form

$$\xrightarrow{R_1 = 2r_1 - r_2} \left(\begin{array}{cc|c} 1 & -10 & -86 \\ 5 & 4 & 56 \end{array}\right)$$

$$\xrightarrow{R_2 = r_2 - 5r_1} \left(\begin{array}{cc|c} 1 & -10 & -86 \\ 0 & 54 & 486 \end{array}\right)$$

$$\xrightarrow{R_2 = R_2/54} \left(\begin{array}{cc|c} 1 & -10 & -86 \\ 0 & 1 & 9 \end{array}\right)$$

- Rewrite the system

$$\begin{cases} u - 10v = -86 \\ v = 9 \end{cases} \Rightarrow u = 10v - 86 = 90 - 86 = 4$$

- $\begin{cases} x^2 = 4 \\ y^2 = 9 \end{cases} \Rightarrow \begin{cases} x = \pm 2 \\ y = \pm 3 \end{cases}$

The system has four solutions $(2,3), (-2,3), (2,-3), (-2,-3)$.

(4) $\begin{cases} 3x - 2y = 6 \\ x + y = 1/2 \end{cases}$

- Augmented matrix $\left(\begin{array}{cc|c} 3 & -2 & 6 \\ 1 & 1 & 1/2 \end{array}\right)$

- Transform it into a row echelon form

$$\xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} 1 & 1 & 1/2 \\ 3 & -2 & 6 \end{array}\right)$$

$$\xrightarrow{R_2 \to R_2 - 3r_1} \left(\begin{array}{cc|c} 1 & 1 & 1/2 \\ 0 & -5 & 9/2 \end{array}\right)$$

$$\xrightarrow{R_2 \to r_2/-5} \left(\begin{array}{cc|c} 1 & 1 & 1/2 \\ 0 & 1 & -9/10 \end{array}\right)$$

- Rewrite the system

$$\begin{cases} x + y = 1/2 \\ \quad\ \ y = -9/10 \end{cases} \Rightarrow x = 1/2 - y = 1/2 + 9/10 = 14/10$$

Therefore, $(x, y) = \left(\frac{14}{10}, \frac{-9}{10}\right) = \left(\frac{7}{5}, \frac{-9}{10}\right)$.