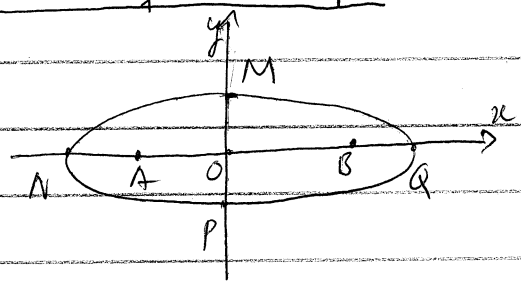
10.3 The ellipse

- Elements of an ellipse
- Solution for problems 17, 27, 39, 43, 55

Elements of an ellipse

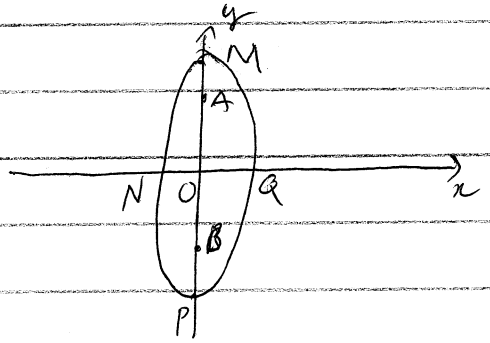


N, Q : vertices

A, B : foci

O : center

major axis : x-axis



M, P : vertices

A, B : foci

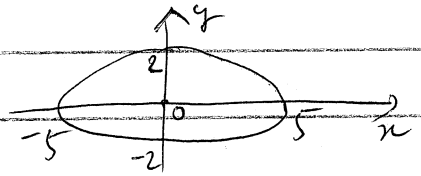
O : center

major axis : y-axis

Problem 17

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

We see that $25 = 5^2$ and $4 = 2^2$. Thus, the shape of the ellipse looks like this:



- Vertices : $(-5,0)$, $(5,0)$
- Major axis : x-axis
- Foci : $c^2 = 5^2 - 2^2 = 21 \Rightarrow c = \sqrt{21} \Rightarrow$ Foci are $(-\sqrt{21}, 0)$, $(\sqrt{21}, 0)$
- Center : $(0,0)$

Problem 27 ∵ Center $(0,0) \Rightarrow \frac{x^2}{\blacksquare^2} + \frac{y^2}{\blacksquare^2} = 1$

∵ focus $(3,0) \Rightarrow$ foci are on x-axis

$\Rightarrow$ major axis is x-axis

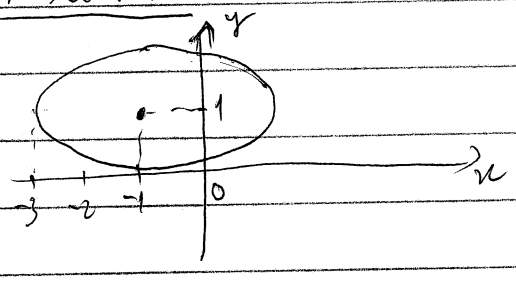
$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a > b$

∵ vertex $(5,0) \Rightarrow a = 5$

$\Rightarrow b^2 = a^2 - c^2 = 5^2 - 3^2 = 16 = 4^2$

Conclusion, $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Problem 39



∵ We see the center $= (-1,1)$

$\Rightarrow \frac{(x+1)^2}{\blacksquare^2} + \frac{(y-1)^2}{\blacksquare^2} = 1$

∵ The major axis is the horizontal line

$\Rightarrow \frac{(x+1)^2}{a^2} + \frac{(y-1)^2}{b^2} = 1, \quad a > b > 0$

∵ The distance from the far west point (vertex) to the center is 2

$\Rightarrow a = 2$

∵ The distance from the far south point to the center is 1

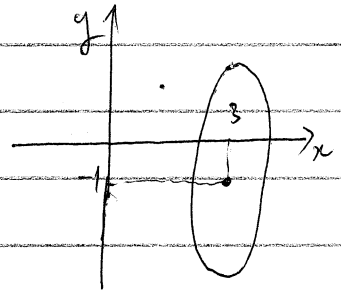
$\Rightarrow b = 1$

Conclusion, $\frac{(x+1)^2}{4} + \frac{(y-1)^2}{1} = 1$

Problem 43 $\quad \frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1$

- We see that the center is $(3, -1)$

- $b = 2$, $a = 3$ $\Rightarrow$ the major axis is ~~y=3~~ $x = 3$
  and vertices are $(3, -1+3) = (3, 2)$,
  $(3, -1-3) = (3, -4)$.

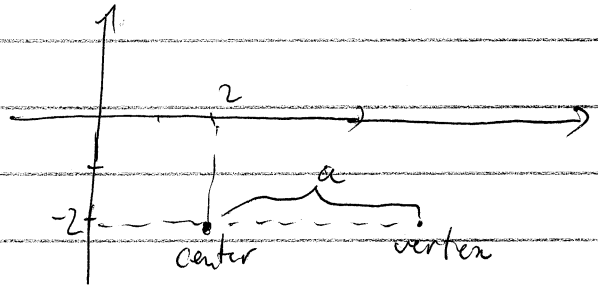


- $c^2 = a^2 - b^2 = 9 - 4 = 5 \Rightarrow c = \sqrt{5}$
  $\Rightarrow$ foci are $(3, -1+\sqrt{5})$ and $(3, -1-\sqrt{5})$

Problem 55

- Center at $(2, -2)$ $\Rightarrow$ $\frac{(x-2)^2}{\blacksquare^2} + \frac{(y+2)^2}{\blacksquare^2} = 1$

- Vertex at $(7, -2)$ $\Rightarrow$ vertices are on a horizontal line



$\Rightarrow \frac{(x-2)^2}{a^2} + \frac{(y+2)^2}{b^2} = 1$ with $a > b > 0$

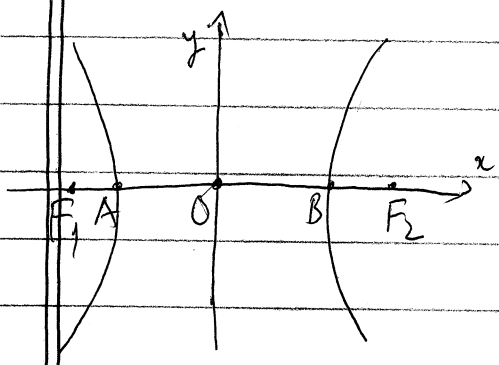
a = distance ($(7,-2)$ and $(2,-2)$) $= 5$

- Focus at $(4, -2)$ $\Rightarrow$ c = distance ($(2,-2)$, and $(4,-2)$) $= 2$
  $\Rightarrow b^2 = a^2 - c^2 = 5^2 - 2^2 = 21$

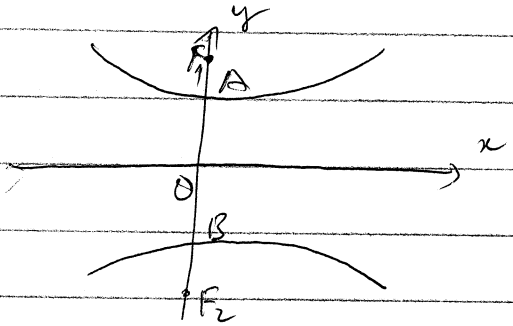
$\Rightarrow \frac{(x-2)^2}{25} + \frac{(y+2)^2}{21} = 1$

10.4 The hyperbola

- Elements of a hyperbola
- Solution for problems 19, 31, 37, 43, 51



- Center: O
- Vertices: A, B
- Foci: F_1, F_2
- ~~Traverse~~ Transverse axis: x-axis
- Conjugate axis: y-axis



- Center: O
- Vertices: A, B
- Foci: F_1, F_2
- Transverse axis: y-axis
- Conjugate axis: x-axis

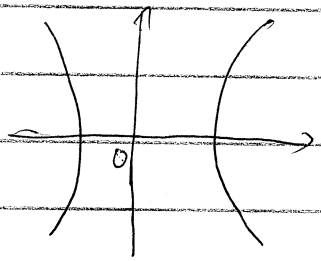
Problem 19 center (0,0), focus (3,0), vertex (1,0)

- Center at (0,0) $\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

- Focus at (3,0) $\Rightarrow$ foci are on the x-axis

$\Rightarrow$ transverse axis is x-axis, conjugate axis y-axis

$\Rightarrow$ the graph look like this:



$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

- Vertex (1,0) $\Rightarrow$ a = distance ((1,0) and (0,0))

$= 1$

- $a^2 + b^2 = c^2 \Rightarrow 1^2 + b^2 = 3^2 \Rightarrow b^2 = 8$

Conclusion $\frac{x^2}{1} - \frac{y^2}{8} = 1$

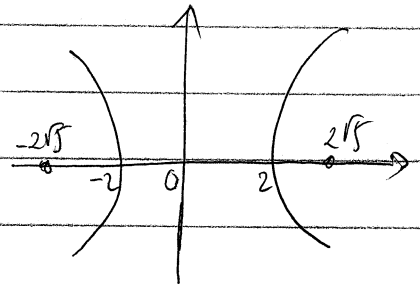
Problem #1, $4x^2 - y^2 = 16$

- Divide both sides by 16, $\frac{x^2}{4} - \frac{y^2}{16} = 1$

$\Rightarrow a^2 = 4$ and $b^2 = 16$

- Center at $(0, 0)$

- Transverse axis is the x-axis, conjugate axis is the y-axis



- Vertices are $(\pm a, 0)$: $(-2, 0)$ and $(2, 0)$

- Foci : $c^2 = a^2 + b^2 = 4 + 16 = 20 \Rightarrow c = \pm 2\sqrt{5}$

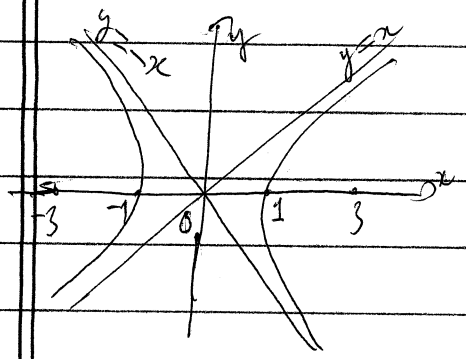
$\Rightarrow$ foci are $(-2\sqrt{5}, 0)$, $(2\sqrt{5}, 0)$

- Asymtotes : $\frac{x^2}{4} - \frac{y^2}{16} = 0$

$\Rightarrow \frac{x}{2} = \pm \frac{y}{4}$

$\Rightarrow y = \pm 2x$

Problem 37



- The transverse axis is the x-axis

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

- Vertices are $(\pm 1, 0) \Rightarrow a = 1$

- Asymptotes: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

$$\Rightarrow \frac{x}{a} = \pm \frac{y}{b}$$

$$\Rightarrow y = \pm \frac{b}{a} x$$

- Look at the picture, see that $\frac{b}{a} = 1 \Rightarrow b = a = 1$

- Conclusion $x^2 - y^2 = 1$

Remark Don't assume by the picture that ± 3 are foci.
(that's just a trap for you)

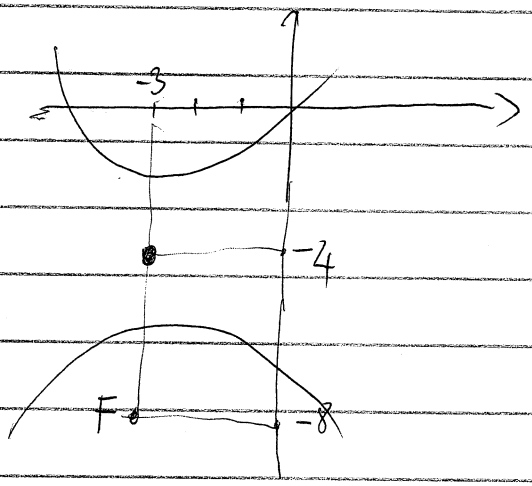
Problem 43 Center $(-3,-4)$, focus $(-3,-8)$, vertex $(-3,-2)$

- Center $(-3,-4) \Rightarrow \dfrac{(x+3)^2}{a^2} - \dfrac{(y+4)^2}{b^2} = 1$ or $\dfrac{(y+4)^2}{a^2} - \dfrac{(x+3)^2}{b^2} = 1$

- focus $(-3,-8)$ $\Rightarrow$ foci are on the vertical line

$\Rightarrow$ transverse axis is vertical

$\Rightarrow \dfrac{(y+4)^2}{a^2} - \dfrac{(x+3)^2}{b^2} = 1$



c = distance $((-3,-4)$ and $(-3,-8))$

$= 5$

- vertex $(-3,-2) \Rightarrow a$ = distance $((-3,-4)$, and $(-3,-2))$

$= 2$

- $a^2 + b^2 = c^2 \Rightarrow 2^2 + b^2 = 5^2 \Rightarrow b^2 = 21$

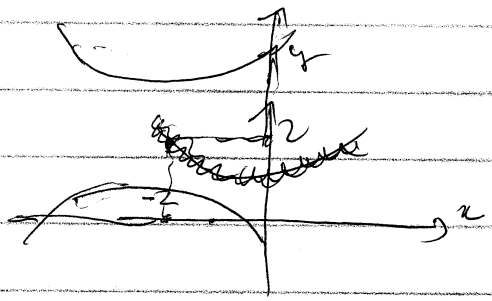
- Conclusion $\dfrac{(y+4)^2}{4} - \dfrac{(x+3)^2}{21} = 1$

Problem 51 $(y-2)^2 - 4(x+2)^2 = 4$

- Divide both sides by 4 : $\frac{(y-2)^2}{4} - \frac{(x+2)^2}{1} = 1$

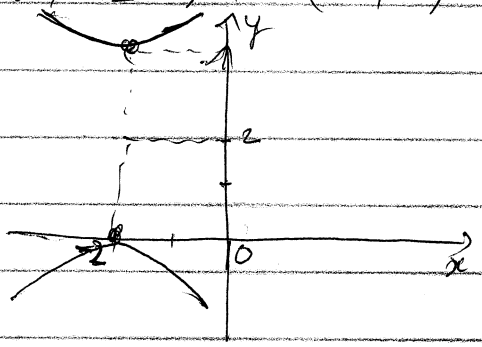
- Center is $(-2, 2)$

- transverse axis is y-axis



- $a^2 = 4$, $b^2 = 1 \Rightarrow c^2 = a^2 + b^2 = 5 \Rightarrow c = \sqrt{5}$

- Vertices are $(-2, 2 \pm a)$: $(-2, 0)$ and $(-2, 4)$



- Foci are $(-2, 2 \pm c)$: $(-2, 2-\sqrt{5})$, and $(-2, 2+\sqrt{5})$

- Asymptotes : $\frac{(y-2)^2}{4} - \frac{(x+2)^2}{1} = 0$

$$\Rightarrow \frac{y-2}{4} = \pm (x+2)$$

* with the plus sign : $\frac{y-2}{4} = x+2 \Rightarrow y-2 = 4(x+2)$

$$\Rightarrow \boxed{y = 4x + 10}$$

* with the minus sign : $\frac{y-2}{4} = -x-2 \Rightarrow y-2 = -4x-8$

$$\Rightarrow \boxed{y = -4x - 6}$$