Main points in Section 6.1

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Only boxed formulas should be memorized!

Contents

1 Convert degrees in decimal to DMS

$$\boxed{1^o = 60'} \tag{1}$$

$$\boxed{1^{'} = 60''} \tag{2}$$

Ex 1: (Exercise 29, page 360)

$$\begin{aligned}
40.32^o &= 40^o + 0.32^o \\
&= 40^o + 0.32 \times 60' \\
&= 40^o + 19.2' \\
&= 40^o + 19' + 0.2' \\
&= 40^o + 19' + 0.2 \times 60'' \\
&= 40^o + 19' + 12'' \\
&= 40^o 19' 12''
\end{aligned}$$

2 Convert degrees in radian to decimal form (and vice versa)

$$\boxed{radian = \frac{\pi}{180} degree} \tag{3}$$

$$\boxed{degree = \frac{180}{\pi} radian} \tag{4}$$

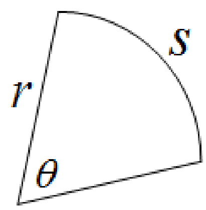
Ex 2: (Exercise 37, page 360)

$$\begin{aligned} radian &= \frac{\pi}{180} degree \\ &= \frac{\pi}{180} \times 240 \\ &= \frac{4\pi}{3} \end{aligned}$$

Ex 3: (Exercise 49, page 360)

$$\begin{aligned} degree &= \frac{180}{\pi} radian \\ &= \frac{180}{\pi}\left(-\frac{5\pi}{4}\right) \\ &= -\frac{180}{\pi}\frac{5\pi}{4} \\ &= -\frac{180 \times 5}{4} \\ &= -225^o \end{aligned}$$

3 Relations among arc's length, radius and angle



$$\boxed{s = r\theta} \tag{5}$$

where θ is in **radian**. Whenever we have two out of three quantities, we can find the other. For example, if we are given s and θ then we can find r by

$$r = \frac{s}{\theta} \tag{6}$$

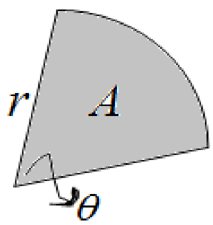
If we are given s and r then we can find θ by

$$\theta = \frac{s}{r} \tag{7}$$

Ex 4: (Exercise 75, page 360)
Using Equation (7), we get

$$\theta = \frac{s}{r} = \frac{3}{5} = 0.6 \ \ \text{radian}$$

4 Relations among sector's area, radius and angle



$$\boxed{A = \frac{1}{2}r^2\theta} \tag{8}$$

where θ is in **radian**. Whenever we have two out of three quantities, we can find the other. For example, if we are given A and r then we can find θ by

$$\theta = \frac{2A}{r^2} \tag{9}$$

If we are given A and θ then we can find r by

$$r = \sqrt{\frac{2A}{\theta}} \tag{10}$$

Ex 5: (Exercise 81, page 360)
Using Equation (10), we get

$$r = \sqrt{\frac{2A}{\theta}} = \sqrt{\frac{2 \times 2}{1/3}} = \sqrt{12} \approx 3.46 \ feet$$

5 Linear speed and angular speed

If an object moves around a circle at constant speed, then

$$\text{Linear speed:} \qquad \boxed{v = \frac{s}{t}} \tag{11}$$

where s is the arc's length that the object has traveled in time t.

$$\text{Angular speed:} \qquad \boxed{\omega = \frac{\theta}{t}} \tag{12}$$

where θ is the angle (in **radian**) that the object has swept out in time t.

6 How to solve word problems ?

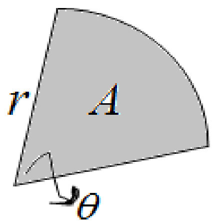
When you are facing a word problem, you can follow the following steps :

1) Draw the picture.

2) Identify what we have (radius, angle, arc's length, sector's area,...) and what we need to find.

3) Try to use the above formulas.

Ex 6 (Problem 94, page 360)
Step 1. Draw the picture :



Step 2. Identify what we have and what we need to find :
$r = 3$ cm
$\theta = 60^o$
$A = ?$
Step 3. Now that we are give the radius and the angle, we will use Equation (8) to find the sector's area :

$$A = \frac{1}{2}r^2\theta$$

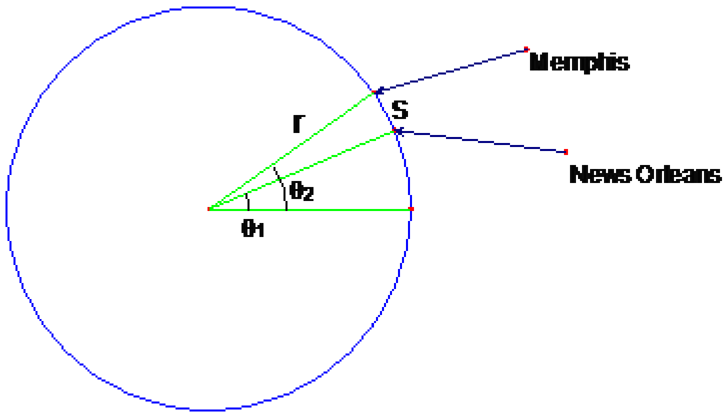
Remember : θ in this formula must be in *radian*. Thus we convert θ in degree to radian :

$$\theta = \frac{\pi}{180} \times 60 = \frac{\pi}{3} \text{ radian}$$

Now we get the sector's area :

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}3^2\frac{\pi}{3} = \frac{3\pi}{2} \approx 4.71\text{cm}^2$$

Ex 7 (Problem 101, page 361)
Step 1. Draw the picture :



Step 2. Identify what we have and what we need to find :
$\theta_1 = 29^o57'$
$\theta_2 = 35^o9'$
$r = 3960$ miles
$s = ?$
Step 3. Here we are given the radius, and asked to find the arc's length. Thus we need to find the central angle. Looking at the picture, we see the central angle is :

$$\theta = \theta_2 - \theta_1 = 35^o9' - 29^o57'$$

What we need is angle θ in *radian*, not in DMS. Thus we should convert θ_2 and θ_1 into decimal degree, and then into radian :

$$\begin{aligned}
\theta_2 &= 35^o + 9' \\
&= 35 + 9 \times \frac{1}{60} \quad \text{degree} \\
&= \left(35 + 9 \times \frac{1}{60}\right) \times \frac{\pi}{180} \quad \text{radian} \\
&\approx 0.6135 \quad \text{radian}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\theta_1 &= 29^o + 57' \\
&= 29 + 57 \times \frac{1}{60} \quad \text{degree} \\
&= \left(29 + 57 \times \frac{1}{60}\right) \times \frac{\pi}{180} \quad \text{radian} \\
&\approx 0.5227 \quad \text{radian}
\end{aligned}$$

Thus $\theta = \theta_2 - \theta_1 \approx 0.6135 - 0.5227 = 0.0908$ radian, and

$$s = r\theta \approx 3960 \times 0.0908 \approx 359.568 \quad \text{miles}$$