Main points in Section 6.2

TA: Tuan Pham

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Contents

1 Important formulas

If we know $\cos\theta$ and $\sin\theta$, we can find the other trigonometric functions :

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}} \tag{1}$$

$$\boxed{\cot\theta = \frac{\cos\theta}{\sin\theta}} \tag{2}$$

$$\boxed{\sec\theta = \frac{1}{\cos\theta}} \tag{3}$$

$$\boxed{\csc\theta = \frac{1}{\sin\theta}} \tag{4}$$

2 Important table of values

θ radians	θ degrees	$\sin\theta$	$\cos\theta$
0	0^0	0	1
$\frac{\pi}{6}$	30^0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{3}$	60^0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{4}$	45^0	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{2}$	90^0	1	0

3 Given the angle. Find the values of 6 trigonometric functions

If you see the angle in the above table, you will know $\cos\theta$ and $\sin\theta$. If the angle is not there, you can follow the following steps :

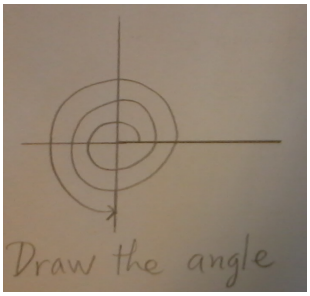
1) Draw the angle to locate the point on unit circle.

2) Locate the position of the point in the unit circle. Try to relate the its position to that of 0^0, 30^0, 45^0, 60^0 or 90^0 by realizing whether they are symmetric with respect to the center, or x-axis, or y-axis.

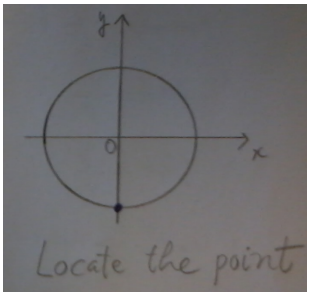
3) Determine x and y, which give us $\cos\theta$ and $\sin\theta$ respectively.

4) Use the formulas (1)-(4) to find other trigonometric functions.

Ex 1 (Problem 25, page 376) Here we are given $\theta = \frac{11\pi}{2}$.
Step 1. Draw the angle :



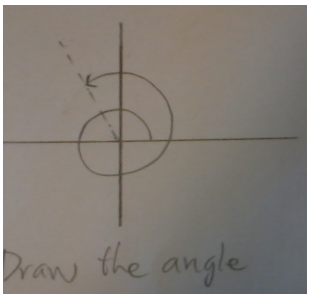
Step 2. Locate the position of the point in the unit circle :



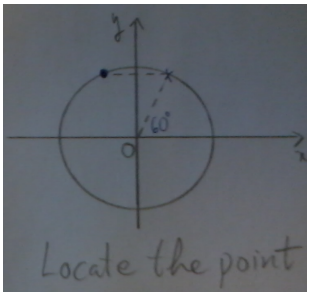
Step 3. We realize that $x = 0$ and $y = -1$. This means $\cos\theta = 0$ and $\sin\theta = -1$.
Step 4. We use fomula (4) to find $\csc\theta$:

$$\csc\frac{11\pi}{2} = \csc\theta = \frac{1}{\sin\theta} = \frac{1}{-1} = -1$$

Ex 2 (Problem 53, page 376) Here we are given $\theta = \frac{8\pi}{3}$.
Step 1. Draw the angle :



Step 2. Locate the position of the point in the unit circle :



We realize that this point and the point of 60^0 are symmetric with respect to the y-axis.
Step 3. We see that x is the negative of the x-value of the point of 60^0, which is $\cos 60^0$. Thus

$$x = -\cos 60^0 = -\frac{1}{2}$$

Also we see that y is exactly the y-value of the point of 60^0, which is $\sin 60^0$. Thus

$$y = \sin 60^0 = \frac{\sqrt{3}}{2}$$

Therefore, now we have

$$\cos\theta = x = -\frac{1}{2}, \quad \sin\theta = y = \frac{\sqrt{3}}{2}$$

Step 4. Now we use formulas (1)-(4) to compute other trigonometric functions :

$$\begin{aligned}
\tan\theta &= \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}}{2}\frac{2}{-1} = -\sqrt{3} \\
\cot\theta &= \frac{\cos\theta}{\sin\theta} = -\frac{1}{2}\frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \\
\sec\theta &= \frac{1}{\cos\theta} = \frac{1}{-\frac{1}{2}} = -2 \\
\csc\theta &= \frac{1}{\sin\theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}
\end{aligned}$$

4 Given (x,y). Find the values of 6 trigonometric functions

If you are given (x, y) and asked to find the values of 6 trigonometric functions, you can follow the following steps :

1) Write $r = \sqrt{x^2 + y^2}$

2) $\cos\theta = \dfrac{x}{r}$ and $\sin\theta = \dfrac{y}{r}$

3) Use the formulas (1)-(4) to find other trigonometric functions.

Ex 3 (Problem 79, page 376)
Here we are given $(x, y) = (2, -3)$.
Step 1. We write

$$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$

Step 2.

$$\begin{aligned}
\cos\theta &= \frac{x}{r} = \frac{2}{\sqrt{13}} \\
\sin\theta &= \frac{y}{r} = \frac{-3}{\sqrt{13}}
\end{aligned}$$

Step 3. Now we use formulas (1)-(4) to compute other trigonometric functions :

$$\begin{aligned}
\tan\theta &= \frac{\sin\theta}{\cos\theta} = \frac{-3}{\sqrt{13}}\frac{\sqrt{13}}{2} = -\frac{3}{2} \\
\cot\theta &= \frac{\cos\theta}{\sin\theta} = \frac{2}{\sqrt{13}}\frac{\sqrt{13}}{-32} = -\frac{2}{3} \\
\sec\theta &= \frac{1}{\cos\theta} = \frac{1}{\frac{2}{\sqrt{13}}} = \frac{\sqrt{13}}{2} \\
\csc\theta &= \frac{1}{\sin\theta} = \frac{1}{\frac{-3}{\sqrt{13}}} = -\frac{\sqrt{13}}{3}
\end{aligned}$$