Main points in Section 6.3

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Contents

1 Important formulas

In this section, we learn 3 more important formulas :

$$\boxed{\sin^2\theta + \cos^2\theta = 1} \tag{1}$$

$$\boxed{\cos^2\theta = \frac{1}{1+\tan^2\theta}} \tag{2}$$

$$\boxed{\cot\theta = \frac{1}{\tan\theta}} \tag{3}$$

Recall that the 4 important formulas in Summary of 6.2 are :

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{4}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \tag{5}$$

$$\sec\theta = \frac{1}{\cos\theta} \tag{6}$$

$$\csc\theta = \frac{1}{\sin\theta} \tag{7}$$

2 Compute trigonometric functions of big angles

A big angle is usually the one that exceeds 2π, or 360^0. We use the periodic properties of trigonometric functions to reduce a big angle to a smaller one, which helps us find these functions more easily. What we need to memorize is : **Only $\tan\theta$ and $\cot\theta$ have period π. The other 4 trigonometric functions have period 2π.**

Ex 1 (Problem 12, page 390)
Here we are asked to find $\cos 420^0$. This is quite a large angle! We know that the function $\cos\theta$ has period 2π, or 360^0. Therefore,

$$\cos 420^0 = \cos(420^0 - 360^0) = \cos(60^0) = \frac{1}{2}$$

Ex 2 (Problem 13, page 390)
Here we are asked to find $\tan 405^0$. This is also a large angle! We know that the function $\tan\theta$ has period π, or 180^0. Therefore,

$$\tan 405^0 = \tan(405^0 - 180^0) = \tan(225^0) = \tan(225^0 - 180^0) = \tan 45^0 = 1$$

3 Given the sign of trigonometric functions. Find the quadrant of θ

If you are given the sign of (usually two) trigonometric functions, and asked to find the quadrant of θ, you can follow the following steps :

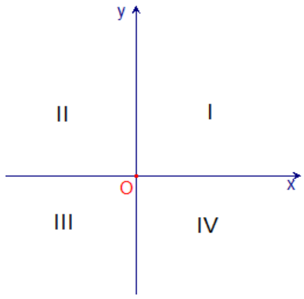
1) Use the formulas (4)-(7) to find the sign of $\cos\theta$ and $\sin\theta$.

2) Draw the coordinate system. By keeping in mind that $\cos\theta$ is x, and $\sin\theta$ is y, you can determine which quadrant θ belongs to.

Ex 3 (Problem 31, page 390)
Here we are given $\cos\theta > 0$ and $\tan\theta < 0$.
Step 1. Try to find the sign of $\cos\theta$ and $\sin\theta$.
We already have the sign for $\cos\theta$. Now look at Formula (4) which relates $\cos\theta$, $\sin\theta$ and $\tan\theta$:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \tag{8}$$

We see that $\sin\theta = \tan\theta\cos\theta < 0$.
Step 2. Now we know that θ belongs to a quadrant of $x > 0$, $y < 0$. This is quadrant IV.



4 Given one trigonometric function and the quadrant of θ. Find other trigonometric functions

If you are given one trigonometric function and the quadrant of θ, and asked to find other trigonometric functions, you can follow the following steps :

1) Look at the quadrant of θ to determine the sign of $\cos\theta$ and $\sin\theta$.

2) Determine $\cos\theta$ and $\sin\theta$. How? If you are given either $\cos\theta$ or $\sin\theta$, you can use Formula (1) to determine the other. If you are given $\tan\theta$, you can use Formula (2) to find $\cos\theta$ and then use Formula (1) to find $\sin\theta$.

3) Use the formulas (1)-(4) to find other trigonometric functions.

<u>Ex 4</u> (Problem 43, page 390)
Here we are given $\sin\theta = \frac{12}{13}$ and θ is in quadrant II.
Step 1. Look at the coordinate system, we see that quadrant II corresponds to $x < 0,\ y > 0$. Hence, $\cos\theta < 0$ and $\sin\theta > 0$.
Step 2. We are already given $\sin\theta = \frac{12}{13}$. Thus, we will use Formula (1) to find $\cos\theta$. We have

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{12}{13}\right)^2 = \frac{25}{169}$$

Because $\cos\theta < 0$, we get

$$\cos\theta = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$$

Step 3. Now we use formulas (1)-(4) to compute other trigonometric functions :

$$\begin{aligned}
\tan\theta &= \frac{\sin\theta}{\cos\theta} = \frac{12}{13}\frac{13}{-5} = -\frac{12}{5} \\
\cot\theta &= \frac{1}{\tan\theta} = -\frac{5}{12} \\
\sec\theta &= \frac{1}{\cos\theta} = -\frac{13}{5} \\
\csc\theta &= \frac{1}{\sin\theta} = \frac{13}{12}
\end{aligned}$$

5 Use even-odd properties to avoid negative angles

If you are asked to find a trigonometric function of a negative angle, and you dislike negative angles, you can use the even-odd properties to avoid it. What you need to memorize is that : **Only $\cos\theta$ and $\sec\theta$ are even. The other 4 tringonometric functions are odd.**

Ex 5 (Problem 61, page 391)

We know that $\tan\theta$ is an odd function. Therefore,

$$\tan(-30^0) = -\tan(30^0) = -\frac{\sqrt{3}}{3}$$