Main points in Section 6.4

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Contents

1 Plot sinusoidal functions

In this section, we learn how to graph sinusoidal functions of the form $y = A\cos(\omega x) + B$ or $y = A\sin(\omega x) + B$. Here are the steps :

1) Remove the negative sign of ω (if it is negative) by using the even-odd properties of sin and cosine. Then we get an equation that looks like

$$y = A\cos(\omega x) + B \quad \text{or} \quad y = A\sin(\omega x) + B$$

but with $\omega > 0$.

2) The amplitude is $|A|$ and the period is $T = \frac{2\pi}{\omega}$.

3) Stick $|A|$ and $-|A|$ on y-axis and draw two horizontal lines from these points; also, stick 0 and T on the x-axis.

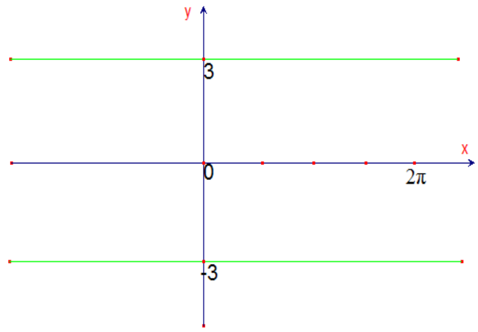
4) Divide the interval $[0, T]$ on x-axis into four equal segments, which gives us five key points.

5) If we are given a sine function, its graph cuts the x-axis at the first, third and fifth points. If we are given a cosine function, its graph cuts the x-axis at the second and fourth points. Ignoring B, we determine the y-values at the rest key points.

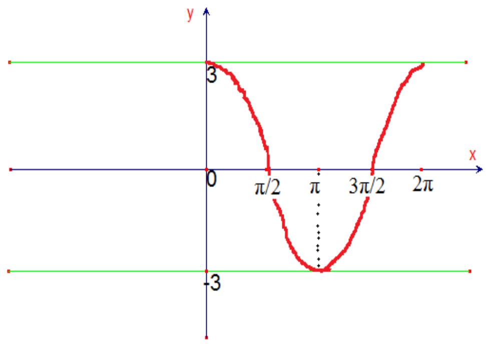
6) Draw the graph in the interval $[0, T]$ and, if you like, extend it periodically.

7) If B is nonzero, we shift the graph up for positive B, or down for negative B.

Ex 1 (Problem 12, page 404) $y = 3\cos x$
Step 1. Here $\omega = 1 > 0$. We skip this step.
Step 2. The amplitude is 3, and the period is $T = \frac{2\pi}{1} = 2\pi$.
Step 3. Stick 3 and -3 on y-axis and draw to horizontal lines from these points; also, stick 0 and 2π on the x-axis.



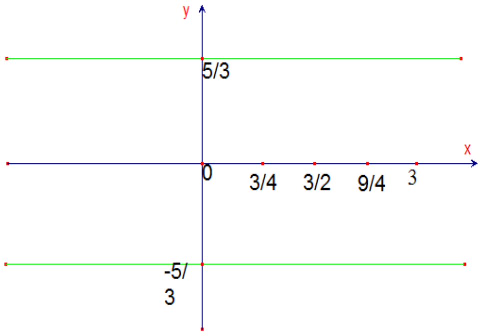
Step 4. The key points are 0, $T/4 = \pi/2$, $T/2 = \pi$, $3T/4 = 3\pi/2$ and $T = 2\pi$. They are red dots in the interval $[0, 2\pi]$ as shown in the above figure.
Step 5. Determine y-values at the key points : we are given a cosine function, so we know that the graph cuts the x-axis at the second and fourth key points. The y-value at 0 is 3. And that is enough for us to draw.
Step 6. Here we draw the graph in only one period. You can extend it periodically if you like.



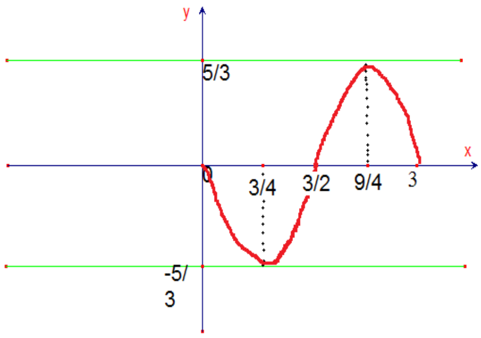
Step 7. Since $B = 0$, we skip this step.
Ex 2 (Problem 19, page 404) $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right)$
Step 1. Here $\omega = -\frac{2\pi}{3} < 0$. We use the odd property of sine function to rewrite the equation as

$$y = -\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$$

Now we have $\omega = \frac{2\pi}{3} > 0$.
Step 2. The amplitude is $\frac{5}{3}$, and the period is $T = \frac{2\pi}{2\pi/3} = 3$.
Step 3. Stick $\frac{5}{3}$ and -$\frac{5}{3}$ on y-axis and draw to horizontal lines from these points; also, stick 0 and 3 on the x-axis.
Step 4. The key points are 0, $T/4 = 3/4$, $T/2 = 3/2$, $3T/4 = 9/4$ and $T = 3$.



They are red dots in the interval $[0, 3]$ as shown in the above figure.
Step 5. Determine y-values at the key points : we are given a sine function, so we know that the graph cuts the x-axis at the first, second and fourth key points. The y-value at 3/4 is -5/3. And that is enough for us to draw.
Step 6. Here we draw the graph in only one period. You can extend it periodically if you like.



Step 7. Since $B = 0$, we skip this step.

2 Determine the equation of a given sinusoidal graph

If you are given a sinusoidal graph and asked to find its equation, you can follow the following steps :

1) Write down the general form

$$y = A\cos(\omega x) + B \tag{1}$$

$$y = A\sin(\omega x) + B \tag{2}$$

2) Determine ω : look at the graph for the period T. Then $\omega = \frac{2\pi}{T}$.

3) Determine the amplitude : $|A|=$ (max-min)/2.

4) Determine B : it is the intersection between the center line of the graph and the y-axis.

5) It B is nonzero, shift the graph up or down B units so that the center of the graph coincide the x-axis.

6) The final step is

- If $y(0) < 0$ then $A < 0$ and the equation of the graph is (1).
- If $y(0) > 0$ then $A > 0$ and the equation of the graph is (1).
- If $y(0) = 0$ and the graph is going up from 0, then $A > 0$ and the equation of the graph is (2).
- If $y(0) = 0$ and the graph is going down from 0, then $A < 0$ and the equation of the graph is (2).

<u>Ex 3</u> (Problem 71, page 406)
Now look at the graph in your book.
Step 1. Write down the general form

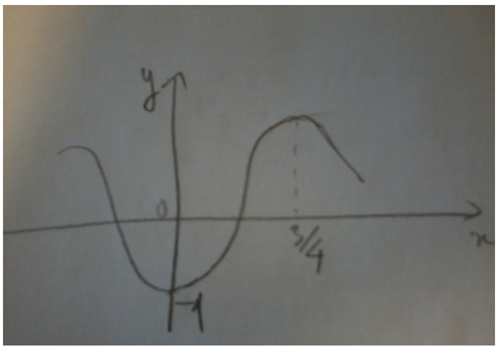
$$y = A\cos(\omega x) + B \tag{3}$$

$$y = A\sin(\omega x) + B \tag{4}$$

Step 2. Determine ω : we see that the period is $T = \frac{3}{2}$. Thus

$$\omega = \frac{2\pi}{T} = 2\pi \times \frac{2}{3} = \frac{4\pi}{3}$$

Step 3. Determine the amplitude : we see that max $= 2$ and min $= 0$. Thus the amplitude is $|A|=$ (max-min)/2=(2-0)/2=1.
Step 4. Determine B : we see that the center line of the graph cut the y-axis at $y = 1$. Therefore $B = 1$.
Step 5. Because $B = 1$ is nonzero, we shift the graph down 1 unit so that the center line of the graph coincide the x-axis.



Step 6. We see that $y(0) = -1 < 0$. Thus $A < 0$ and $A = -1$. The equation of the graph is (3) :

$$y = -\cos(\frac{4\pi}{3}x) + 1 \tag{5}$$