

Please show all necessary work completely and clearly.

- (1) (7 Points) Determine whether the function $y(x) = xe^x$ is a solution to the initial value problem

$$y'(x) - y(x) = e^x, \quad y(0) = 0.$$

Answer: Yes. We have that $y'(x) = e^x + xe^x$ and therefore $y'(x) - y(x) = [e^x + xe^x] - xe^x = e^x$, so the differential equation is satisfied.

We also have to check the initial condition $y(0) = 0$. Indeed, $y(0) = 0 * e^0 = 0$.

2 points for any work

2 points for computing y'

2 points for comparing $y' - y$ to e^x

1 point for checking the initial condition

—

7 points total

- (2) (3 Points) Suppose $y(x)$ is a solution to the differential equation

$$\frac{dy}{dx} = x^2 + xy + y^2.$$

If the point $(1, -1)$ is on the graph of $y(x)$, then what is $y'(1)$?

Answer: We have to evaluate $\frac{dy}{dx}$ at $(x, y) = (1, -1)$. We have

$$\frac{dy}{dx} = x^2 + xy + y^2 = 1^2 + 1(-1) + (-1)^2 = 1 - 1 + 1 = \mathbf{1}.$$

1 point for any work

1 point for plugging $(1, -1)$ into $x^2 + xy + y^2$

1 point for correct answer

—

3 points total