Please show all necessary work completely and clearly.

(1) (7 Points) Determine whether the function $y(x) = xe^x$ is a solution to the initial value problem
$$y'(x) - y(x) = e^x, \qquad y(0) = 0.$$

(2) (3 Points) Suppose $y(x)$ is a solution to the differential equation
$$\frac{dy}{dx} = x^2 + xy + y^2.$$
If the point $(1, -1)$ is on the graph of $y(x)$, then what is $y'(1)$?