

(1) (7 points) Given the following autonomous differential equation

$$\frac{dx}{dt} = x^2(x^2 - 1).$$

Find all critical points. Draw the phase diagram and indicate whether each critical point is stable or unstable.

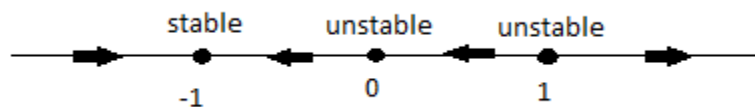
- Factor the right hand side: 1 point

$$x^2(x - 1)(x + 1)$$

- Get correct critical points: 1 point

$$c = -1, 0, 1.$$

- Draw correct arrows: 2 points



- Indicate correctly stability/unstability: 3 points (1 point for each critical point).

(2) (3 points) Given the following initial value problem

$$\frac{dy}{dx} = x \cos(y), \quad y(0) = 1$$

Write the iterative formula of Euler's method and the initial point. (You can denote the step size by h .)

Correct answer: get full credit.

$$y_{n+1} = y_n + hx_n \cos(y_n), \quad x_0 = 0, \quad y_0 = 1.$$

If not, get partial credit as follows.

- Being able to write $x_0 = 0, y_0 = 1$: 1 point
- Being able to write the left hand side as $\frac{y_{n+1} - y_n}{h}$: 2 points