Math 2243 Fall 2015 - Quiz 7
11/12/2015 - 10 Minutes **Name:**

Please show all necessary work completely and clearly.

(1) (5 Points) Show that $y(x) = 1, y(x) = e^{-x}$, and $y(x) = e^{2x}$ are linearly independent functions.

Answer: We compute the Wronskian of the three functions, which is the determinant of the following matrix:

$$\begin{pmatrix} 1 & e^{-x} & e^{2x} \\ 0 & -e^{-x} & 2e^{2x} \\ 0 & e^{-x} & 4e^{2x} \end{pmatrix}$$

The determinant should equal $-6e^{x}$, which students should recognize is not the zero function. Therefore, the functions are linearly independent.

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Give 3/5 or 4/5 points for those who use the correct method but have algebra mistakes, either in calculating the derivatives or calculating the determinant.

Students may also attempt an argument beginning with "Suppose $c_1(1)+c_2(e^{-x})+c_3(e^{2x}) = 0.''$ Probably they will not get far with this, but give 1 or 2 points in this case.

I think as far as partial credit goes, these will be most of the cases.

(2) (5 Points) Find the general solution to the differential equation

$$y''' - y'' - 2y' = 0.$$

Answer: $c_1 + c_2e^{-x} + c_3e^{2x}$.

One approach is to recognize that the three functions in problem 1 are solutions. Since they are linearly independent, and there are three of them for a degree 3 differential equation, then the general solution is $c_1 + c_2e^{-x} + c_3e^{2x}$.

A student may check that the three functions are solutions but not make a conclusion about the general solution. 3/5 for this.

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A second method is via the characteristic equation $\lambda^3 - \lambda^2 - 2\lambda = 0$. The solutions to this are $\lambda = 0, -1, 2$, which correspond to the solutions $y = 1, y = e^{-x}, y = e^{2x}$. Thus the general solution is $c_1 + c_2e^{-x} + c_3e^{2x}$. In this case:

2 points for writing down the characteristic equation

1 point for getting the correct roots

1 point for recognizing the solutions $1, e^{-x}, e^{2x}$

1 point for correct general solution