

Find general solutions of the following system. The eigenvalues of the coefficient matrix A are given.

$$\vec{x}' = \begin{bmatrix} 2 & 0 & 0 \\ -7 & 9 & 7 \\ 0 & 0 & 2 \end{bmatrix} \vec{x}, \quad \lambda = 2, 2, 9$$

(3 points) Find two eigenvectors for $\lambda = 2$. For example,

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

Give at least 1 point if the student shows effort to solve a linear system.

(3 points) Find an eigenvector for $\lambda = 1$. For example,

$$\vec{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Give at least 1 point if the student shows effort to solve a linear system.

(3 points) Get 1 point for each correct function

$$\vec{x}_1(t) = e^{2t}\vec{v}_1, \quad \vec{x}_2(t) = e^{2t}\vec{v}_2, \quad \vec{x}_3(t) = e^{9t}\vec{v}_3$$

Give at least 2 points if the answers are wrong but the forms are correct.

(1 point) Get $\vec{x}(t) = c_1\vec{x}_1(t) + c_2\vec{x}_2(t) + c_3\vec{x}_3(t)$