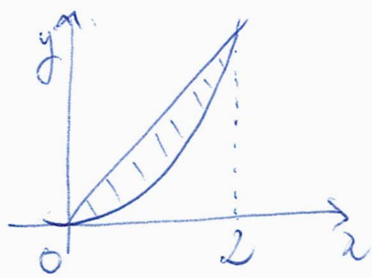
Name___________________________________ December 6, 2016

Quiz 10

1. Sketch the region enclosed by the curves $y = x^2$ and $y = 2x$. Then find the area of the region.

2. Let r and h be positive numbers. Find the volume of the solid obtained by rotating the region bounded by the curves $x = r\left(1 - \frac{y}{h}\right)$, $y = 0$, $x = 0$ about the y-axis. *(Note: This is a way to derive the volume of a cone of altitude h and radius r.)*

①



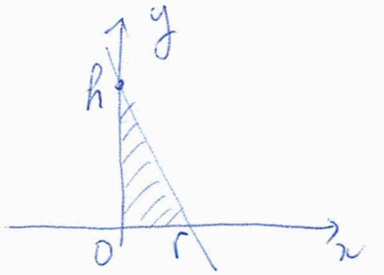
At the intersection point:

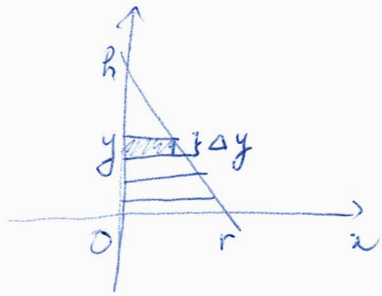
$x^2 = 2x \quad \Rightarrow x = 0$ or 2

The area of the region is

$$\int_0^2 (2x - x^2)\,dx = \left(x^2 - \frac{x^3}{3}\right)_0^2 = \frac{4}{3}$$

②





Approximate the region by horizontal rectangles. The flat cylinder obtained by rotating each rectangle about the y-axis has volume

$$V(y) = \pi x^2 \Delta y = \pi \left[r\left(1 - \frac{y}{h}\right)\right]^2 \Delta y$$

Summing all these volumes and letting $\Delta y \to 0$,

$$\text{Volume} = \int_0^h \pi \left[r\left(1 - \frac{y}{h}\right)\right]^2 dy$$

Put $u = 1 - \frac{y}{h}$. Then $du = -\frac{1}{h}\,dy$, or $dy = -h\,du$.

$$\text{Volume} = \int_1^0 \pi(ru)^2(-h)\,du = \pi r^2 h \int_0^1 u^2\,du = \frac{\pi r^2 h}{3}.$$