Math 1271 September 20, 2016

Quiz 2

1. At what points is $f(x)$ continuous? Explain why.

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ e^x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x > 1 \end{cases}$$

2. Find the following limits (support your answers with calculation)

(a)

$$\lim_{x\to\infty} \frac{2x^2}{3x^2+x+1}$$

(b)

$$\lim_{x\to\infty} \frac{x}{x^2+1}$$

① $f(x)$ is continuous at every point in $(-\infty,0)\cup(0,1)\cup(1,\infty)$.

Now check whether $f(x)$ is continuous at 0 and 1.

$$\left.\begin{aligned}\lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-}(x+2) = 0+2=2\\ \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+} e^x = e^0 = 1\end{aligned}\right\} \Rightarrow \lim_{x\to 0} f(x) \text{ DNE}$$

$$\left.\begin{aligned}\lim_{x\to 1^-} f(x) &= \lim_{x\to 1^-} e^x = e^1 = e\\ \lim_{x\to 1^+} f(x) &= \lim_{x\to 1^+}(2-x) = 2-1=1\end{aligned}\right\} \Rightarrow \lim_{x\to 1} f(x) \text{ DNE}$$

Thus, $f(x)$ is discontinuous at 0 and 1. We conclude that all the points at which f is continuous are

$$(-\infty,0)\cup(0,1)\cup(1,\infty).$$

② (a) $$\lim_{x\to\infty}\frac{2x^2}{3x^2+x+1} = \lim_{x\to\infty}\frac{2}{3+\frac{1}{x}+\frac{1}{x^2}} = \frac{2}{3+0+0} = \frac{2}{3}$$

(b) $$\lim_{x\to\infty}\frac{x}{x^2+1} = \lim_{x\to\infty}\frac{\frac{1}{x}}{1+\frac{1}{x^2}} = \frac{0}{1+0} = 0.$$