Name_____ October 25, 2016

# Quiz 6

1. Find the absolute maximum and absolute minimum values of the function

$$f(x) = 2x^3 - 3x^2 + 6 \quad \text{on} \quad [-1, 2].$$

2. Show that the following equation has exactly one root

$$3x + \sin x + \cos x = 0.$$

(1) $f'(x) = 6x^2 - 6x = 6x(x-1)$

Critical points are $x = 0$ and $x = 1$.

$f(-1) = 2(-1)^3 - 3(-1)^2 + 6 = 1$

$f(0) = 6$

$f(1) = 5$

$f(2) = 10$

The absolute maximum of $f$ on $[-1, 2]$ is $f(2) = 10$.

The absolute minimum of $f$ on $[-1, 2]$ is $f(-1) = 1$.

(2) Put $f(x) = 3x + \sin x + \cos x$.

$f$ is continuous on $\mathbb{R}$.

$f(-1) = -3 + \sin(-1) + \cos(-1) < 0$

$f(1) = 3 + \sin 1 + \cos 1 > 0$

By the Intermediate Value Theorem, there exists $c \in (-1, 1)$ such that $f(c) = 0$.

$f'(x) = 3 + \cos x - \sin x > 0$

Thus, $f$ is an increasing function. It, therefore, has at most one root. We conclude that $f$ has exactly one root.