Background Check

This is not a placement test, and it will not count toward your grade, but it will help you identify topics in the prerequisite mathematics that may give you trouble and make it difficult to succeed in Calculus I. It is not a complete review of background topics. Performing well on these problems does not guarantee success in the course.
For more practice on the prerequisite mathematics for Calculus I, see the **diagnostic tests on page xxiv** and **Appendices A-D** in the textbook.

1. Given $f(x) = x^3$, and $h \neq 0$, first expand the expression $f(x+h)$. Then write

$$\frac{f(x+h) - f(x)}{h}$$

   in terms of x and h, simplifying as much as possible.

2. Rewrite the following with no square roots in the denominator:

$$\frac{x}{\sqrt{x+2} - \sqrt{2x}}$$

3. (a) Sketch in the xy-plane the set of ordered pairs of real numbers (x, y) that satisfy the equation
$$x^2 + 6x + 8y + y^2 = 0.$$

   (b) Find all ordered pairs of real numbers (x, y) that satisfy both equations:
$$x^2 + 6x + 8y + y^2 = 0 \quad \text{and} \quad x - 2y = 0.$$

4. (a) Find all real numbers x that satisfy the equation
$$\sin(2x) = \frac{\sqrt{3}}{2}.$$
   Angles are measured in radians.

   (b) Find all real numbers x that satisfy the equation
$$1 - e^{x^2-5} = 0.$$

① $f(x) = x^3$

$$f(x+h) = (x+h)^3 = (x+h)^2(x+h) = (x^2+2xh+h^2)(x+h) = x^3+3x^2h+3xh^2+h^3$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^3-x^3}{h} = \frac{\cancel{x^3}+3x^2h+3h^2x+h^3-\cancel{x^3}}{h}$$

$$= \boxed{3x^2+3hx+h^2}$$

②
$$\frac{x}{\sqrt{x+2}-\sqrt{2x}} = \frac{x\left(\sqrt{x+2}+\sqrt{2x}\right)}{\left(\sqrt{x+2}-\sqrt{2x}\right)\left(\sqrt{x+2}+\sqrt{2x}\right)}$$

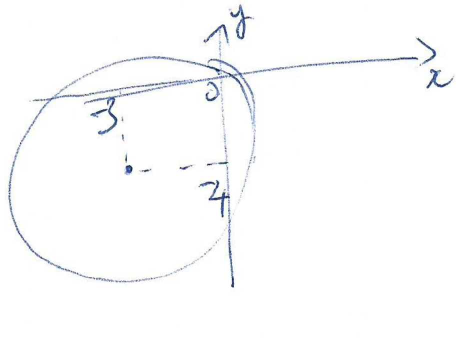
$$= \frac{x\left(\sqrt{x+2}+\sqrt{2x}\right)}{\sqrt{x+2}^2-\sqrt{2x}^2}$$

$$= \frac{x\left(\sqrt{x+2}+\sqrt{2x}\right)}{2-x}$$

③ The equation can be rewritten as

$$(x+3)^2+(y+4)^2 = 5^2$$

This is the equation of the circle of radius 5 centered at $(x,y) = (-3,-4)$.



Intersection with the line $x-2y=0$: the coordinates of the intersection point satisfy

$$\begin{cases} x^2+6x+8y+y^2=0 \\ x-2y=0 \end{cases}$$

From the second equation, $x=2y$. Substituting this x into the first equation, we get $5y^2+20y=0$, which gives $y = 0$ or $y = -4$. Therefore, the two ~~intect~~ intersection points are $(0,0)$ and $(-8,-4)$.

(4) $$\sin(2x) = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$$

Thus, $$2x = \frac{\pi}{3} + k2\pi \quad \text{or} \quad \left(\pi - \frac{\pi}{3}\right) + k2\pi \quad (k \in \mathbb{Z})$$

Therefore, $$x = \frac{\pi}{6} + k\pi \quad \text{or} \quad \frac{\pi}{3} + k\pi \quad (k \in \mathbb{Z})$$

(5) $$1 - e^{x^2-5} = 0$$

implies $$1 = e^{x^2-5}$$

Taking the natural logarithm of both sides, we get

$$0 = x^2 - 5$$

which gives $x = \pm\sqrt{5}$.